筑波大学集中講義2025

原子核理論特講II 「ハンズオンで学ぶ核力の基礎」

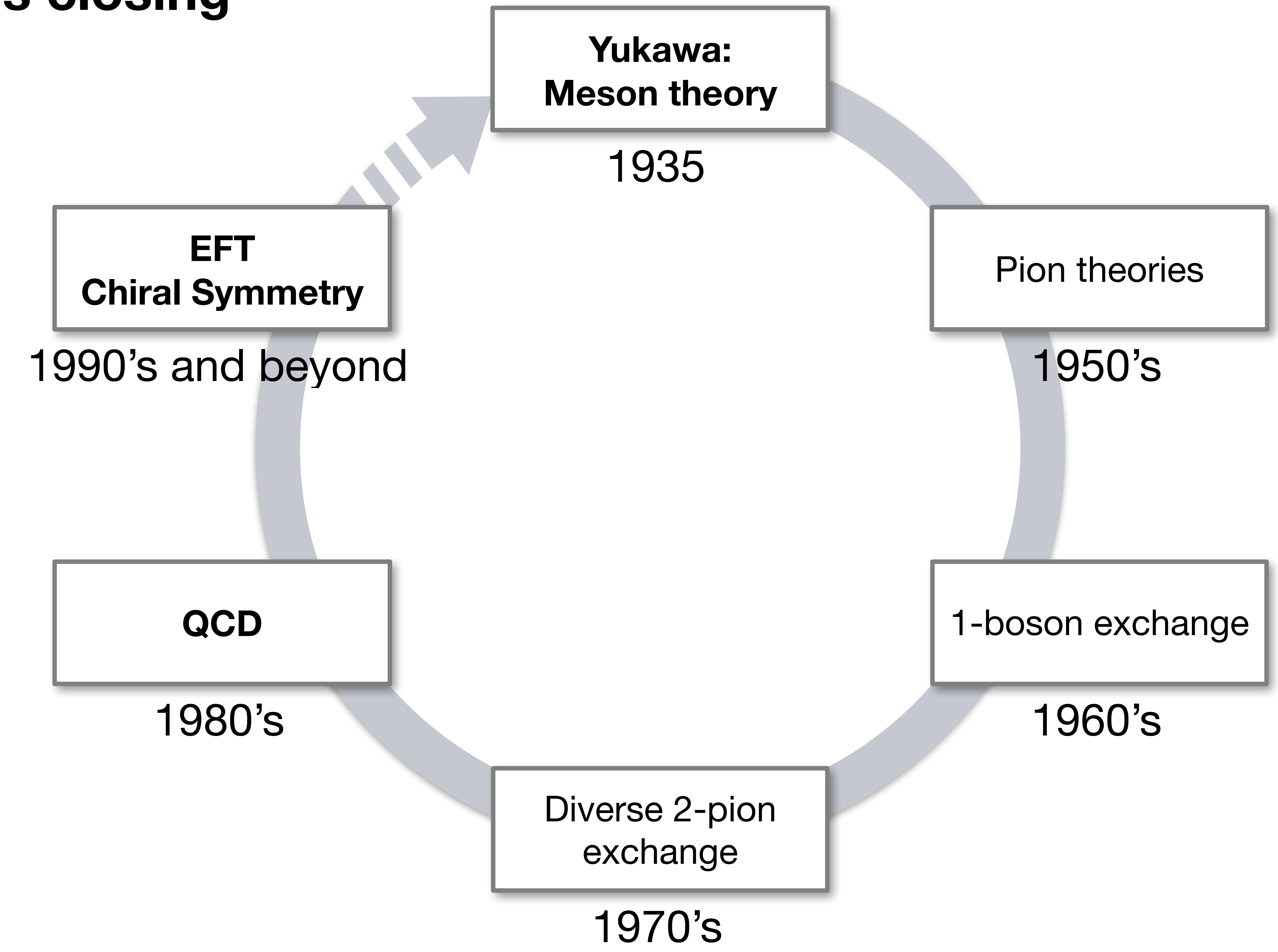
— 第六回「カイラル有効場理論と最近の話題」 —

Brief introduction of chiral EFT

History of nuclear-force study

“the circle of history is closing”

Machleidt & Entem, PR **503**, 1 (2011)



Concept of chiral EFT

Chiral effective field theory (EFT)

Weinberg, PA **96**, 327 (1979)
Epelbaum +, RMP 81, 1773 (2009)
Machleidt & Entem, PR **503**, 1 (2011)

Degrees of freedom and symmetry

Nucleons and pions
Chiral symmetry

→

Soft scale: $Q \sim m_\pi$
Hard scale: $\Lambda_\chi \sim m_\rho$

Perturbative expansion of Lagrangian

$\left(\frac{Q}{\Lambda_\chi}\right)^\nu$ Power counting
Theoretical uncertainty

$$\mathcal{L}_{\rm eff} = \mathcal{L}_\pi + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \cdots$$

$$\mathcal{L}_\pi = \frac{F^2}{4}\mathrm{Tr}(\boldsymbol{\nabla}^\mu U \boldsymbol{\nabla}_\mu U^\dagger + \chi_+) + \cdots$$

$$\mathcal{L}_{\pi N} = \bar{N}(iv \cdot D + g_A \cdot S)N + \cdots$$

$$\mathcal{L}_{NN} = -\frac{1}{2}C_S(\bar{N}N)^2 + 2C_T(\bar{N}SN)^2 + \cdots$$

Many-body forces on an equal footing

Leading 3NF at N²LO ($n = 3$)

More details

Lagrangian

Weinberg, PA **96**, 327 (1979)
Epelbaum +, RMP 81, 1773 (2009)
Machleidt & Entem, PR **503**, 1 (2011)

$$\mathcal{L}_{\rm eff} = \mathcal{L}_\pi + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} + \cdots$$

$$\mathcal{L}_\pi^{(0)} = \frac{F^2}{4}\langle \nabla^\mu U \nabla_\mu U^\dagger + \chi_+\rangle,$$

$$\mathcal{L}_{\pi N}^{(0)} = \bar{N}(iv\cdot D + \mathring{g}_A u\cdot S)N,$$

$$\mathcal{L}_{NN}^{(0)} = -\tfrac{1}{2}C_S(\bar{N}N)(\bar{N}N) + 2C_T(\bar{N}SN)\cdot(\bar{N}SN),$$

$$\begin{aligned}\mathcal{L}_\pi^{(2)} = {}& \frac{l_3}{16}\langle\chi_+\rangle^2 + \frac{l_4}{16}[2\langle\nabla_\mu U\nabla^\mu U^\dagger\rangle\langle\chi_+\rangle \\ &+ 2\langle\chi^\dagger U\chi^\dagger U + \chi U^\dagger\chi U^\dagger\rangle - 4\langle\chi^\dagger\chi\rangle - \langle\chi_-\rangle^2] \\ &+ \cdots ,\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\pi N}^{(2)} = \bar{N}\bigg(&\frac{1}{2\mathring{m}}(v\cdot D)^2 - \frac{1}{2\mathring{m}}D\cdot D + d_{16}S\cdot u\langle\chi_+\rangle \\ &+ id_{18}S^\mu[D_\mu,\chi_-] + \cdots\bigg)N,\end{aligned}$$

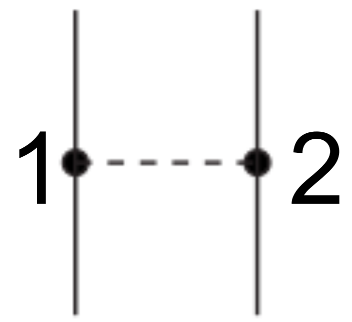
$$\begin{aligned}\mathcal{L}_{NN}^{(2)} = {}& -\tilde{C}_1\{(\bar{N}DN)\cdot(\bar{N}DN) + [(D\bar{N})N]\cdot[(D\bar{N})N]\} \\ &- 2(\tilde{C}_1 + \tilde{C}_2)(\bar{N}DN)\cdot[(D\bar{N})N] \\ &- \tilde{C}_2(\bar{N}N)\cdot[(D^2\bar{N})N + \bar{N}D^2N] + \cdots\end{aligned}$$

Nucleon field	N
Pion field	$\boldsymbol{\pi}$
Nucleon four-velocity	v_μ
Covariant spin vector	$S_\mu \equiv (1/2)i\gamma_5\sigma_{\mu\nu}v^\nu$
Pion decay constant	F
Axial vecor coupling const.	$\mathring{g}_A$
Low-energy constant	C_S and C_T
Unitary 2×2 matrix	$U(\boldsymbol{\pi}) = u^2(\boldsymbol{\pi})$ $U(\boldsymbol{\pi}) = 1 + \frac{i}{F}\boldsymbol{\tau}\cdot\boldsymbol{\pi} - \frac{1}{2F^2}\boldsymbol{\pi}^2 + \mathcal{O}(\pi^3)$
Covariant derivative (nucleon)	$D_\mu = \partial_\mu + [u^\dagger, \partial_\mu u]/2$
Covariant derivative (pion)	$u_\mu = i(u^\dagger\partial_\mu u - u\partial_\mu u^\dagger)$
Chiral symmetry breaking	$\chi_+ = u^\dagger\chi u^\dagger + u\chi^\dagger u$ $\quad \chi = 2B\mathcal{M}$
Finite light quark masses	$\mathcal{M} = \mathrm{diag}(m_u, m_d)$
Scalar quark condensate	$\langle 0\vert\bar{u}u\vert 0\rangle = -F^2B$
Pion-quark-mass relation	$M_\pi^2 = 2Bm_q + \mathcal{O}(m_q^2)$

Power counting

Courtesy: T. Miyagi (Tsukuba)

- Examples: $\nu = -4 + 2N + 2L + \sum_i \Delta_i, \;\; \Delta_i = d_i + \frac{n_i}{2} - 2$

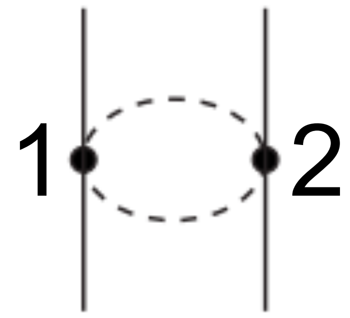


$N = 2, L = 0, \Delta_1 = \Delta_2 = 0$

$\nu = 0$

$N = 2, L = 0, \Delta = 0$

$\nu = 0$

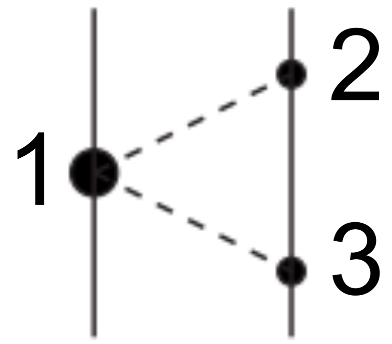


$N = 2, L = 1, \Delta_1 = \Delta_2 = 0$

$\nu = 2$

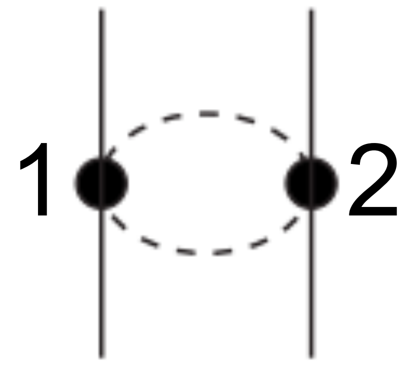
$N = 2, L = 0, \Delta = 2$

$\nu = 2$



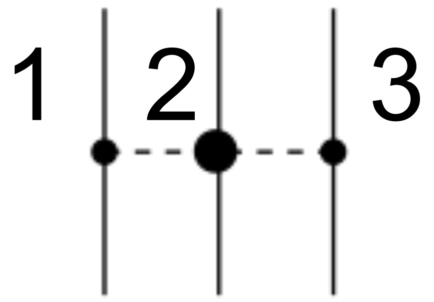
$N = 2, L = 1, \Delta_1 = 1, \Delta_2 = \Delta_3 = 0$

$\nu = 3$



$N = 2, L = 1, \Delta_1 = \Delta_2 = 1$

$\nu = 4$



$N = 3, L = 0, \Delta_1 = \Delta_3 = 0, \Delta_2 = 1$

$\nu = 3$

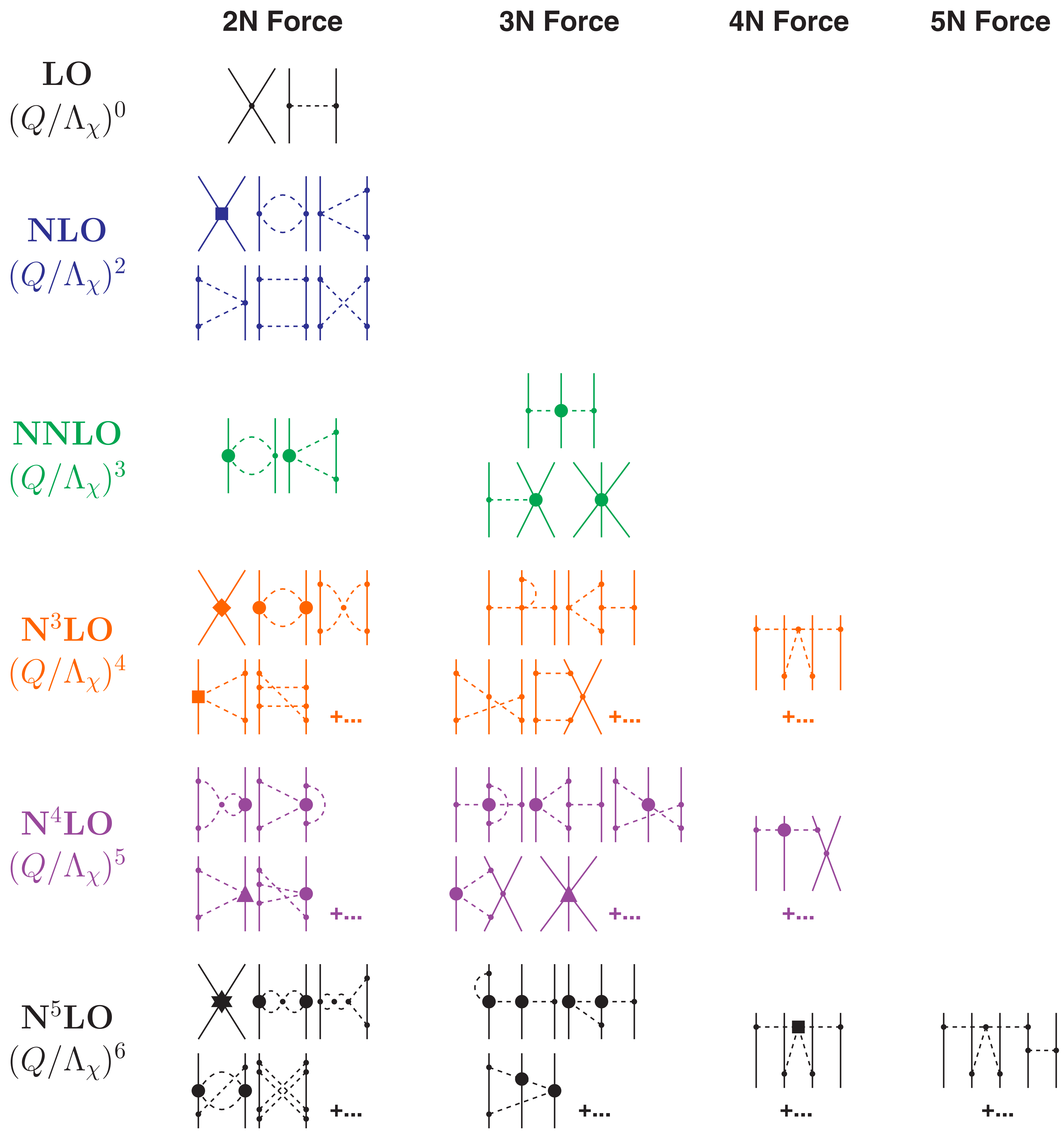
dot : $\Delta = 0$

circle : $\Delta = 1$

square : $\Delta = 2$

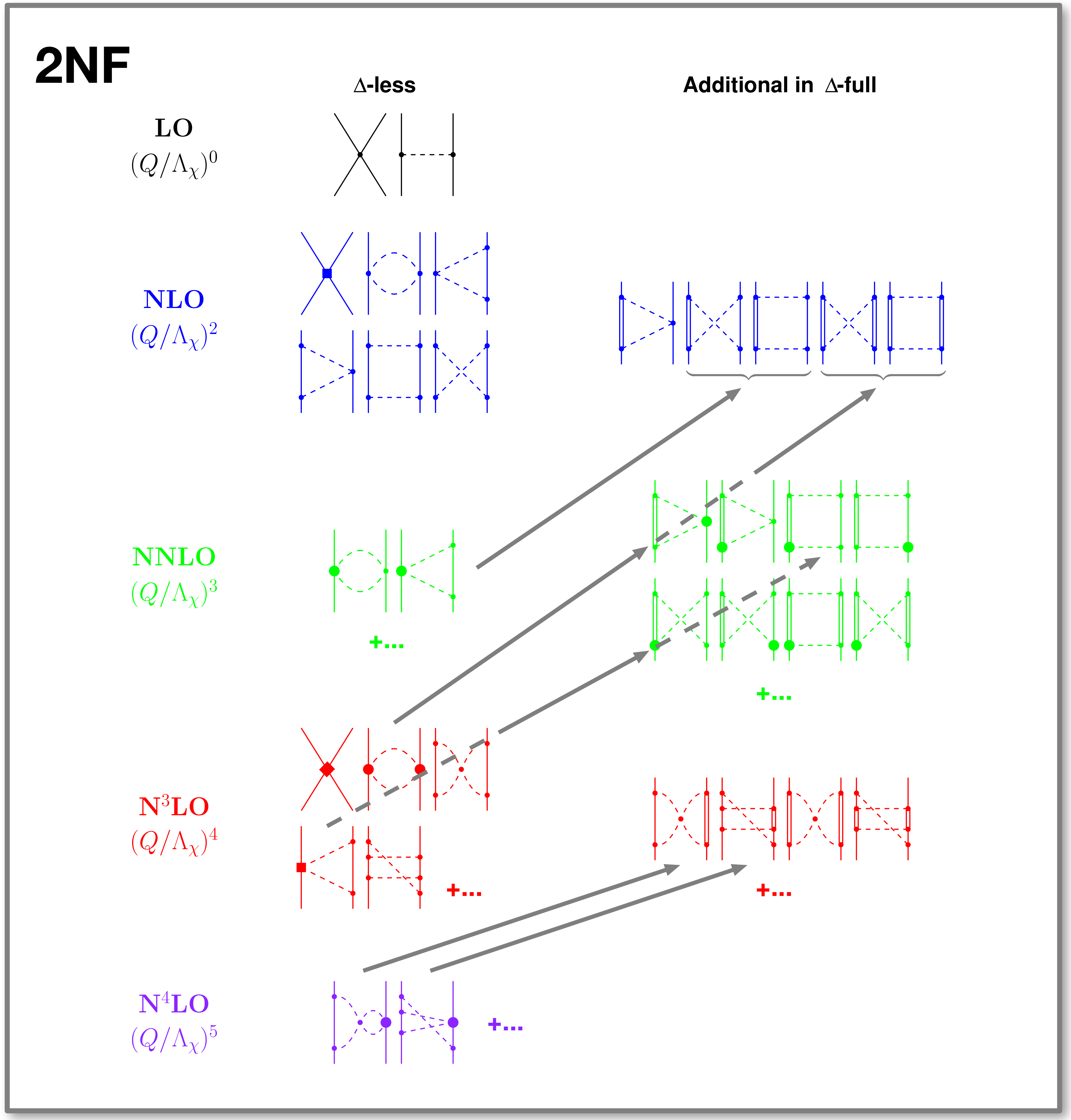
Hierarchy of chiral EFT

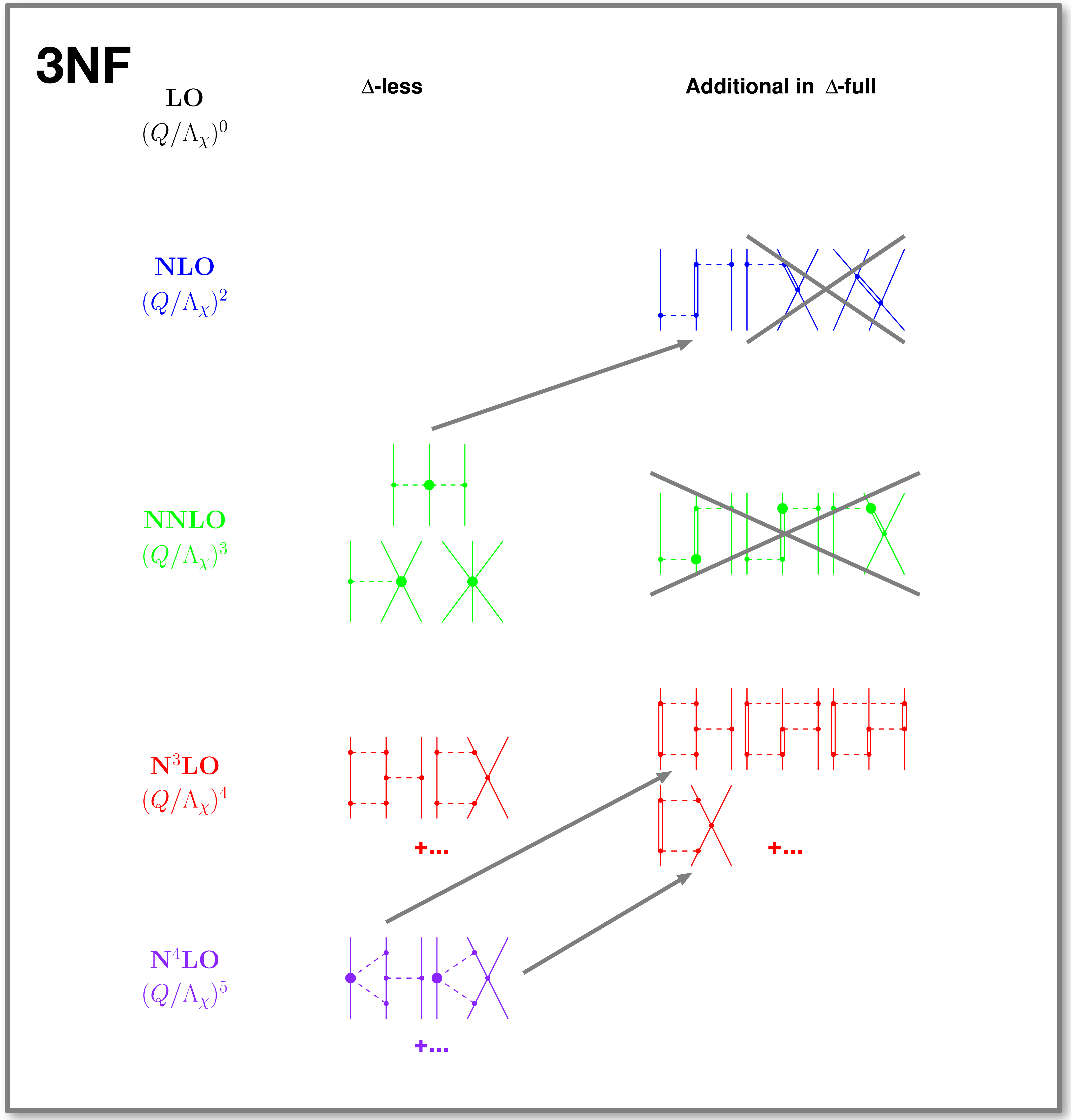
Entem +, PRC **96**, 024004 (2017)



Hierarchy of Δ-full chiral EFT

Machleidt & Entem, PR **503**, 1 (2011)





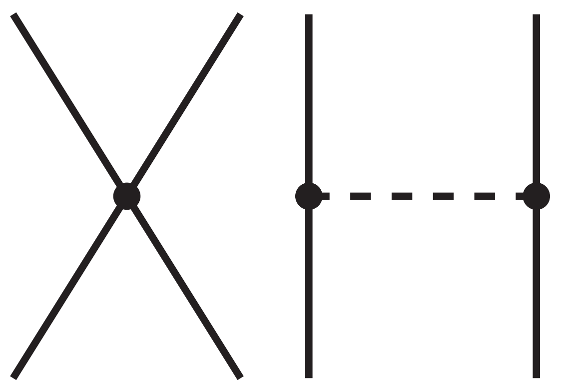
LO potential

Machleidt & Entem, PR **503**, 1 (2011)

$$\hat{V}_{2N}^{(\mathrm{LO})} = \hat{V}_{1\pi} + \hat{V}_{\mathrm{ct}}^{(0)}$$

LO

$(Q/\Lambda_\chi)^0$



$$v_{1\pi}(\boldsymbol{p}', \boldsymbol{p}) = -\left(\frac{g_A}{2f_\pi}\right)^2 \frac{(\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})\,(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q})}{q^2 + m_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

Central + Tensor

Momentum transfer

$$\boldsymbol{q} = \boldsymbol{p}' - \boldsymbol{p}$$

$$v_{\mathrm{ct}}^{(0)}(\boldsymbol{p}', \boldsymbol{p}) = C_S + C_T\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

Central

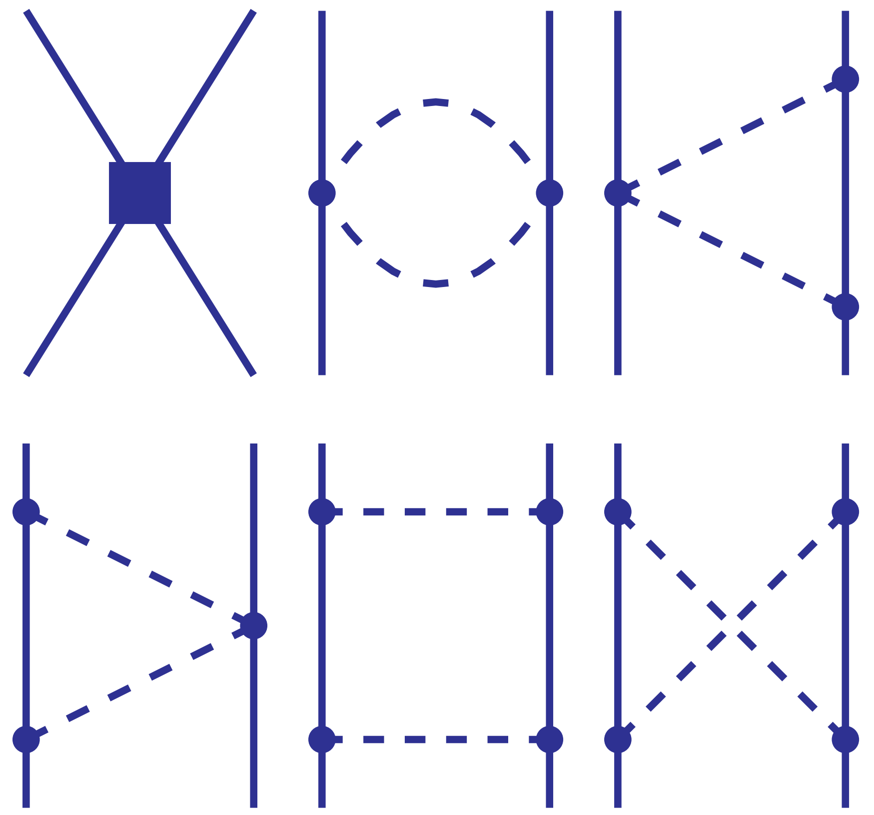
NLO potential

Machleidt & Entem, PR **503**, 1 (2011)

$$\hat{V}_{2N}^{(\mathrm{NLO})} = \hat{V}_{2\pi}^{(2)} + \hat{V}_{\mathrm{ct}}^{(2)}$$

NLO

$(Q/\Lambda_\chi)^2$



$$v_{2\pi}^{(2)}(\boldsymbol{p}',\boldsymbol{p}) = W_C^{(2)}(q,Q)\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + V_S^{(2)}(q,Q)\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + V_T^{(2)}(q,Q)\,(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})\,(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})$$

Central + Tensor

$$v_{\mathrm{ct}}^{(2)}(\boldsymbol{p}',\boldsymbol{p}) = C_1 q^2 + C_2 Q^2 + \left(C_3 q^2 + C_4 Q^2\right)\boldsymbol{\sigma}_a\cdot\boldsymbol{\sigma}_b + C_5\left[-i\boldsymbol{S}\cdot(\boldsymbol{q}\times\boldsymbol{Q})\right] + C_6\,(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})\,(\boldsymbol{\sigma}_2\cdot\boldsymbol{q}) + C_7\,(\boldsymbol{\sigma}_1\cdot\boldsymbol{Q})\,(\boldsymbol{\sigma}_2\cdot\boldsymbol{Q})$$

Central + SO + Tensor

Average momentum

$$\boldsymbol{Q} = (\boldsymbol{p}' + \boldsymbol{p})/2$$

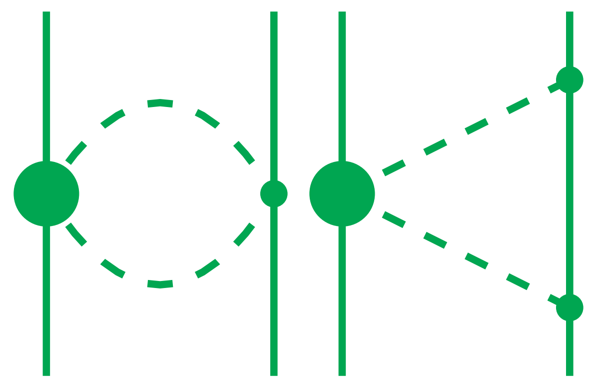
N²LO potential

Machleidt & Entem, PR **503**, 1 (2011)

$$\hat{V}_{2N}^{(\mathrm{N^2LO})} = \hat{V}_{2\pi}^{(3)}$$

NNLO

$(Q/\Lambda_\chi)^3$



$$\begin{aligned} v_{2\pi}^{(3)}(\boldsymbol{p}', \boldsymbol{p}) = {} & V_C^{(3)}(q,Q) + W_C^{(3)}(q,Q)\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \\ & + \left[V_S^{(3)}(q,Q) + W_S^{(3)}(q,Q)\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \right] \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \\ & + \left[V_{LS}^{(3)}(q,Q) + W_{LS}^{(3)}(q,Q)\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \right] \left[-i \boldsymbol{S} \cdot (\boldsymbol{q} \times \boldsymbol{Q}) \right] \\ & + \left[V_T^{(3)}(q,Q) + W_T^{(3)}(q,Q)\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \right] (\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})\, (\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}) \,. \end{aligned}$$

Central + SO + Tensor

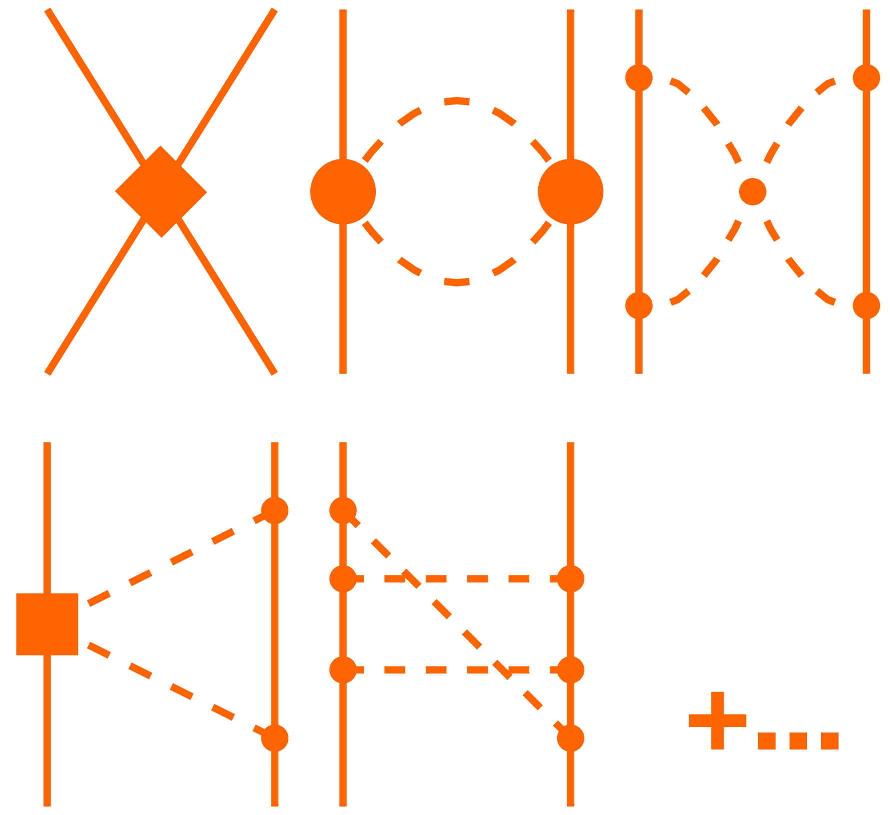
N³LO potential

Machleidt & Entem, PR **503**, 1 (2011)

$$\hat{V}_{2N}^{(\mathrm{N^3LO})} = \hat{V}_{2\pi}^{(4)} + \hat{V}_{\mathrm{ct}}^{(4)}$$

N^{3}LO

$(Q/\Lambda_\chi)^4$



$$\begin{aligned}
v_{\mathrm{ct}}^{(4)}(\boldsymbol{p}',\boldsymbol{p}) &= D_1 q^4 + D_2 Q^4 + D_3 q^2 Q^2 + D_4 (\boldsymbol{q}\times\boldsymbol{Q})^2 \\
&+ \left[D_5 q^4 + D_6 Q^4 + D_7 q^2 Q^2 + D_8 (\boldsymbol{q}\times\boldsymbol{Q})^2\right] \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 \\
&+ \left[D_9 q^2 + D_{10} Q^2\right]\left[-i\boldsymbol{S}\cdot(\boldsymbol{q}\times\boldsymbol{Q})\right] \\
&+ \left[D_{11} q^2 + D_{12} Q^2\right](\boldsymbol{\sigma}_1\cdot\boldsymbol{q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{q}) \\
&+ \left[D_{13} q^2 + D_{14} Q^2\right](\boldsymbol{\sigma}_1\cdot\boldsymbol{Q})(\boldsymbol{\sigma}_2\cdot\boldsymbol{Q}) \\
&+ D_{15}\left[\boldsymbol{\sigma}_1\cdot(\boldsymbol{q}\times\boldsymbol{Q})\right]\left[\boldsymbol{\sigma}_2\cdot(\boldsymbol{q}\times\boldsymbol{Q})\right]
\end{aligned}$$

Central + SO + Tensor

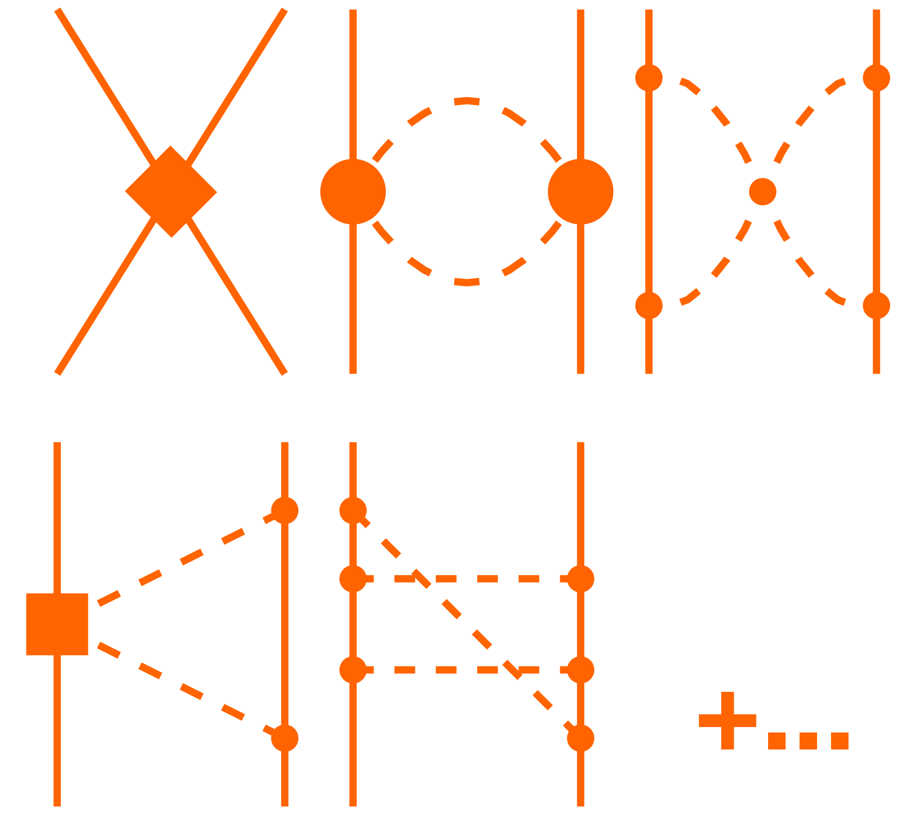
N³LO potential

Machleidt & Entem, PR **503**, 1 (2011)

$$\hat{V}_{2N}^{(\mathrm{N^3LO})} = \hat{V}_{2\pi}^{(4)} + \hat{V}_{\mathrm{ct}}^{(4)}$$

N^{3}LO

$(Q/\Lambda_\chi)^4$



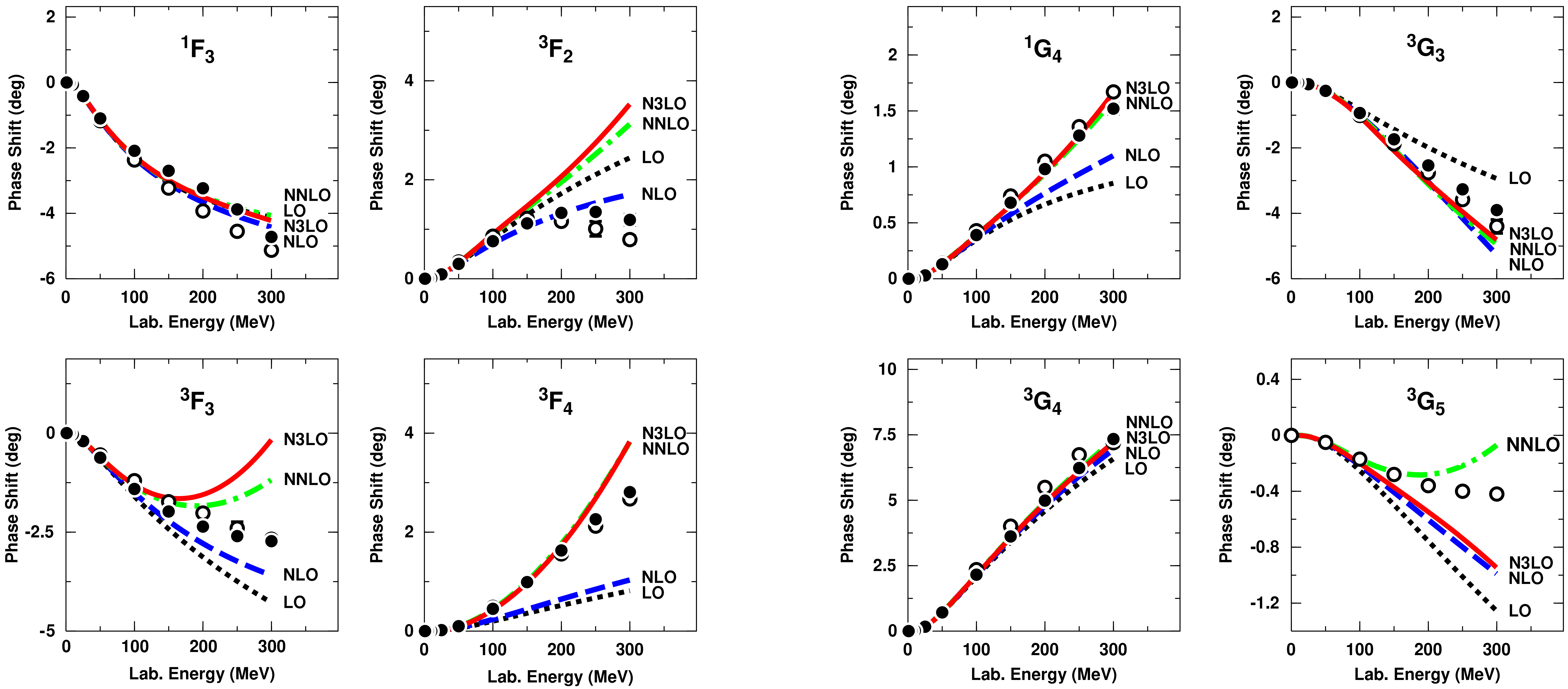
$$\begin{aligned}
v_{2\pi}^{(4)}(\boldsymbol{p}',\boldsymbol{p}) = {} & V_C^{(c_i^2)}(q,Q) + W_S^{(c_i^2)}(q,Q)\,(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\,(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) + W_T^{(c_i^2)}(q,Q)\,(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})\,(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})\,(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \\
& + V_C^{(c_i/m_N)}(q,Q) + W_C^{(c_i/m_N)}(q,Q)\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + W_S^{(c_i/m_N)}(q,Q)\,(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\,(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \\
& + \left[V_{LS}^{(c_i/m_N)}(q,Q) + W_{LS}^{(c_i/m_N)}(q,Q)\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right]\left[-i\boldsymbol{S}\cdot(\boldsymbol{q}\times\boldsymbol{Q})\right] \\
& + W_T^{(c_i/m_N)}(q,Q)\,(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})\,(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})\,(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2) \\
& + V_C^{(m_N^{-2})}(q,Q) + W_C^{(m_N^{-2})}(q,Q)\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \\
& + \left[V_S^{(m_N^{-2})}(q,Q) + W_S^{(m_N^{-2})}(q,Q)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right]\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 \\
& + \left[V_{LS}^{(m_N^{-2})}(q,Q) + W_{LS}^{(m_N^{-2})}(q,Q)\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right]\left[-i\boldsymbol{S}\cdot(\boldsymbol{q}\times\boldsymbol{Q})\right] \\
& + \left[V_T^{(m_N^{-2})}(q,Q) + W_T^{(m_N^{-2})}(q,Q)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right](\boldsymbol{\sigma}_1\cdot\boldsymbol{q})\,(\boldsymbol{\sigma}_2\cdot\boldsymbol{q}) \\
& + V_{\sigma L}^{(m_N^{-2})}(q,Q)\left[\boldsymbol{\sigma}_1\cdot(\boldsymbol{q}\times\boldsymbol{Q})\right]\left[\boldsymbol{\sigma}_2\cdot(\boldsymbol{q}\times\boldsymbol{Q})\right] \\
& + V_C^{(\mathrm{2L})}(q,Q) + W_C^{(\mathrm{2L})}(q,Q)\,\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \\
& + \left[V_S^{(\mathrm{2L})}(q,Q) + W_S^{(\mathrm{2L})}(q,Q)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right](\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2) \\
& + \left[V_T^{(\mathrm{2L})}(q,Q) + W_T^{(\mathrm{2L})}(q,Q)\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right](\boldsymbol{\sigma}_1\cdot\boldsymbol{q})\,(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})
\end{aligned}$$

Central + SO + Tensor

NN-phase shift by chiral EFT at N³LO

Peripheral interaction ($1\pi + 2\pi$)

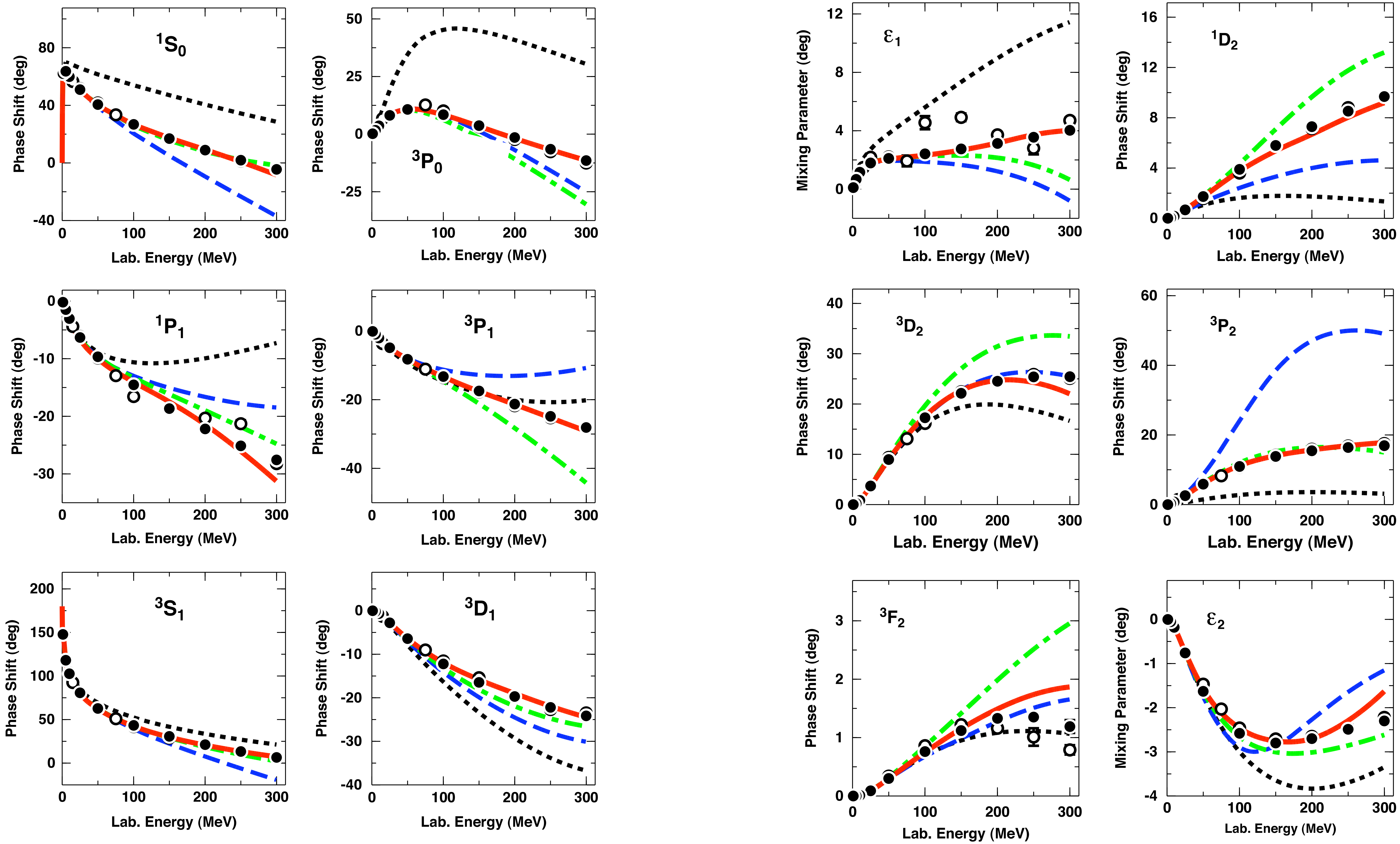
Machleidt & Entem, PR **503**, 1 (2011)



NN-phase shift by chiral EFT at N³LO

Full interaction (peripheral + contact)

Machleidt & Entem, PR **503**, 1 (2011)



NN-phase shift by chiral EFT at N⁴LO

Order-by-order convergence

Entem +, PRC **96**, 024004 (2017)

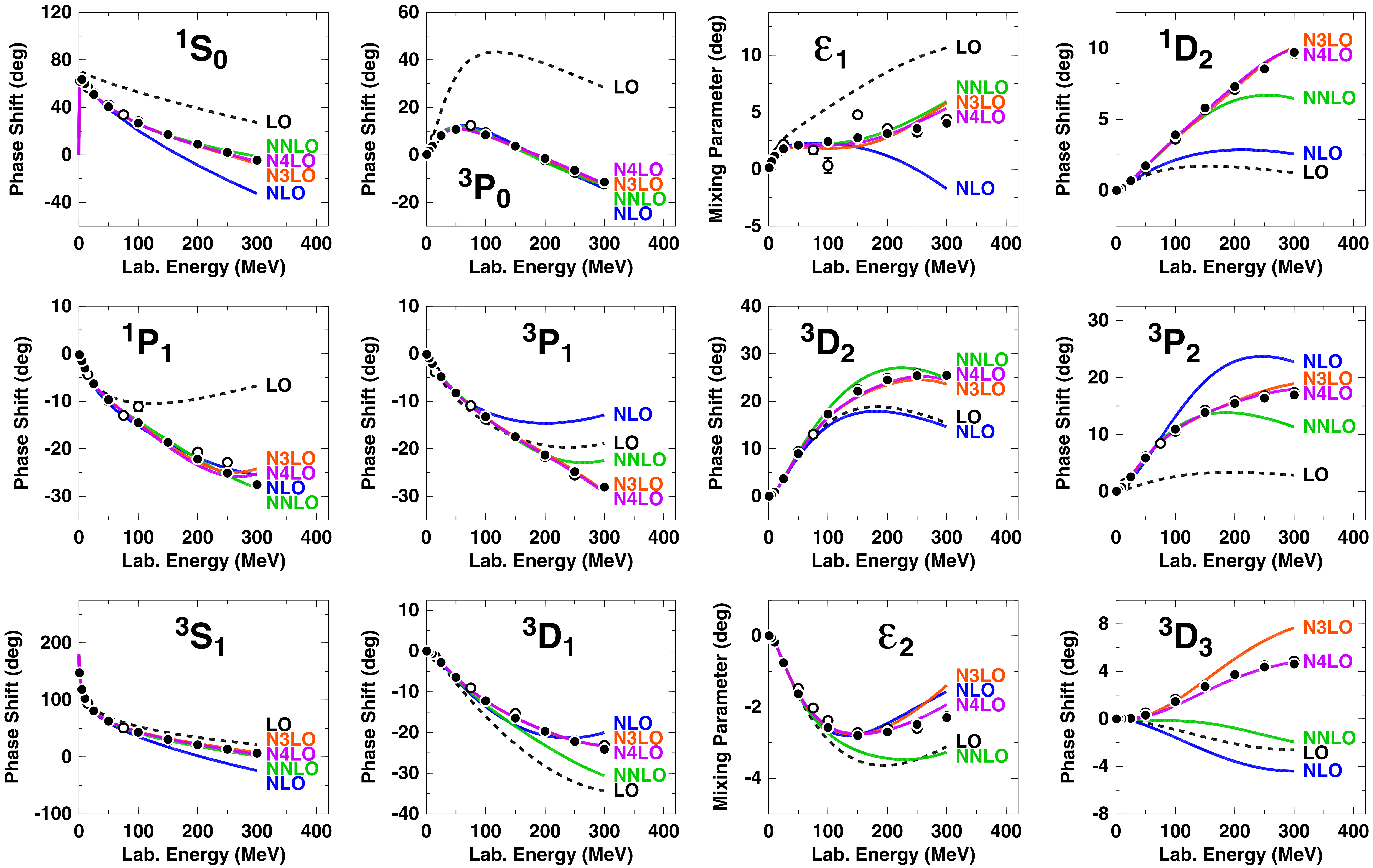
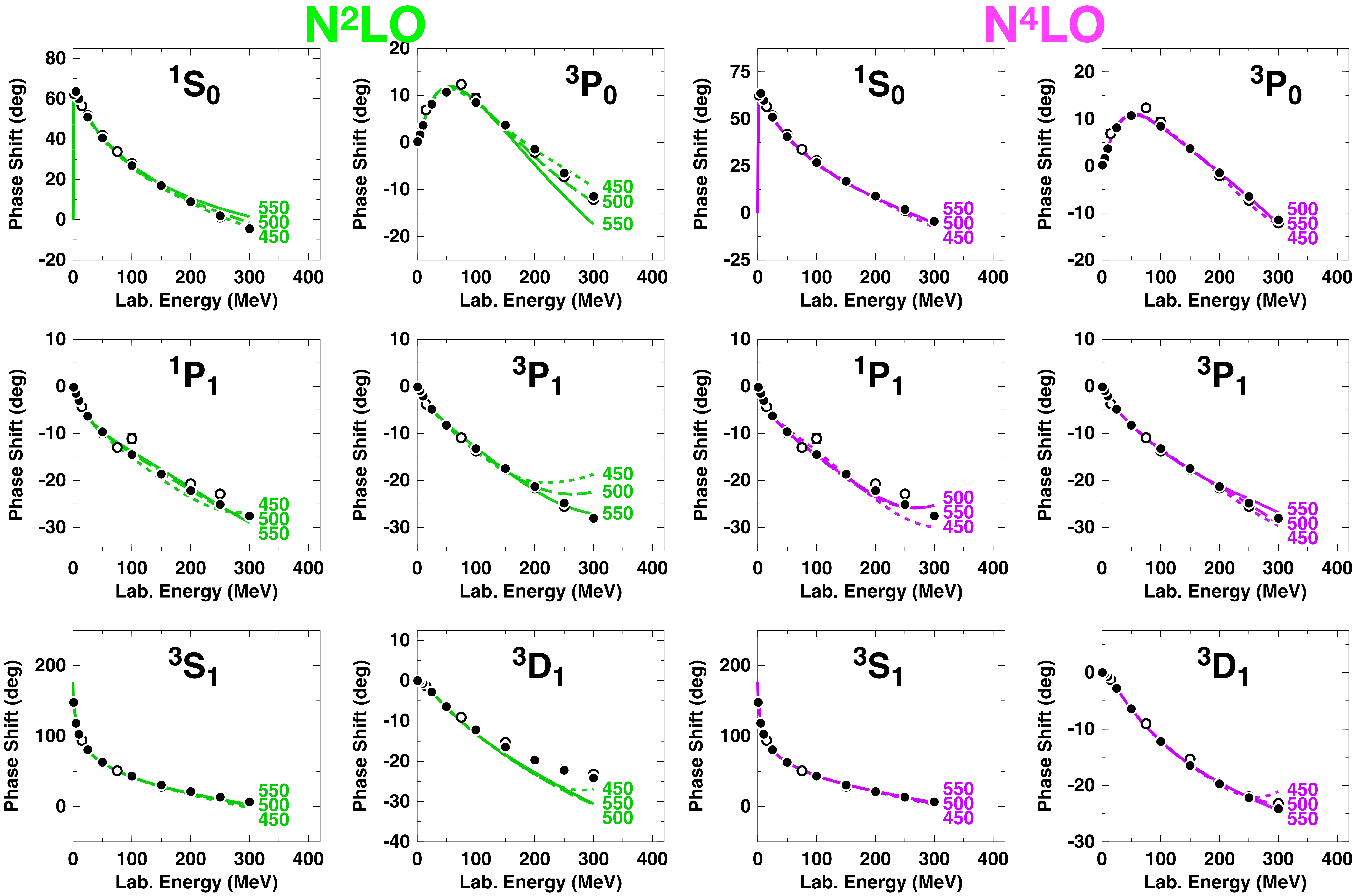


FIG. 2. Chiral expansion of neutron-proton scattering as represented by the phase shifts in S, P, and D waves and mixing parameters ϵ_1 and ϵ_2. Five orders ranging from LO to N^{4}LO are shown as denoted. A cutoff $\Lambda = 500$ MeV is applied in all cases. The filled and open circles represent the results from the Nijmegen multienergy *np* phase-shift analysis [80] and the GWU single-energy *np* analysis SP07 [102], respectively.

NN-phase shift by chiral EFT at N^4LO

Cutoff dependence

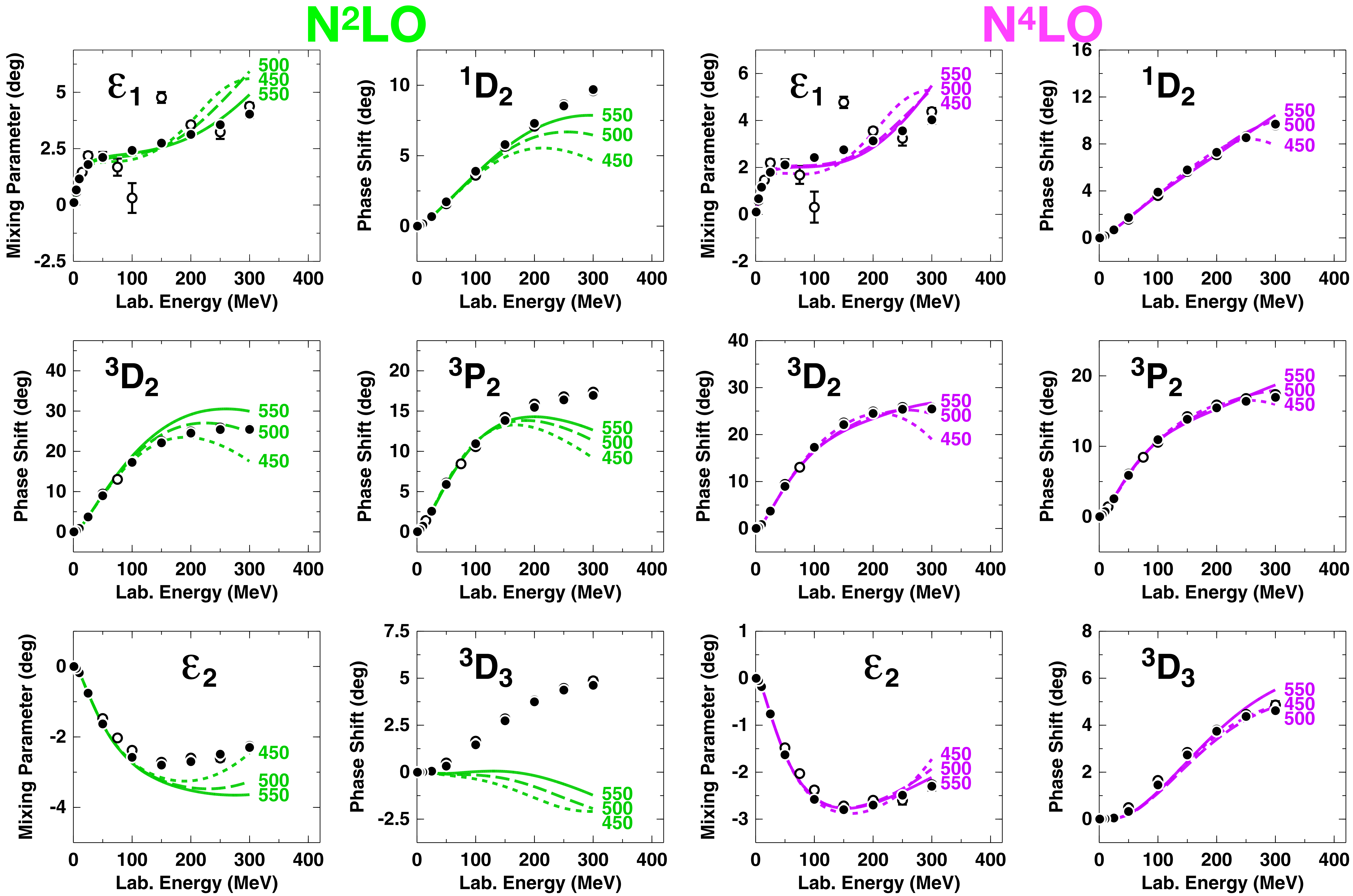
Entem +, PRC **96**, 024004 (2017)



NN-phase shift by chiral EFT at N⁴LO

Cutoff dependence

Entem +, PRC **96**, 024004 (2017)



χ^2 and summary

Machleidt & Entem, PR **503**, 1 (2011)

Entem +, PRC **96**, 024004 (2017)

Columns three to five display the χ^2/datum for the reproduction of the 1999 ***np* database** [173] (subdivided into energy intervals) by various *np* potentials. For the chiral potentials, the χ^2/datum is stated in terms of ranges which result from a variation of the cutoff parameters used in the regulator functions. The values of these cutoff parameters in units of MeV are given in parentheses. $T_{\rm lab}$ denotes the kinetic energy of the incident nucleon in the laboratory system.

$T_{\rm lab}$ bin (MeV)	# of **np** data	Idaho N^3LO [68] (500–600)	Juelich N^3LO [171] (600/700–450/500)	Argonne V_{18} [174]
0–100	1058	1.0–1.1	1.0–1.1	0.95
100–190	501	1.1–1.2	1.3–1.8	1.10
190–290	843	1.2–1.4	2.8–20.0	1.11
0–290	2402	1.1–1.3	1.7–7.9	1.04

TABLE VIII. χ^2/datum for the fit of the *pp* plus *np* data up to 190 MeV and two- and three-nucleon bound-state properties as produced by *NN* potentials at NNLO and N^4LO applying different values for the cutoff parameter Λ of the regulator function Eq. (2.43). For some of the notation, see Table VII, where also empirical information on the deuteron and triton can be found.

Λ(MeV)	NNLO			N^4LO		
	450	500	550	450	500	550
χ^2/datum *pp* and *np*						
0–190 MeV (2903 data)	4.12	3.27	3.32	1.17	1.08	1.25
Deuteron						
B_d (MeV)	2.224575	2.224575	2.224575	2.224575	2.224575	2.224575
A_S (fm$^{-1/2}$)	0.8847	0.8844	0.8843	0.8852	0.8852	0.8851
η	0.0255	0.0257	0.0258	0.0254	0.0258	0.0257
$r_{\rm str}$ (fm)	1.967	1.968	1.968	1.966	1.973	1.971
Q (fm^2)	0.269	0.273	0.275	0.269	0.273	0.271
P_D (%)	3.95	4.49	4.87	4.38	4.10	4.13
Triton						
B_t (MeV)	8.35	8.21	8.10	8.04	8.08	8.12

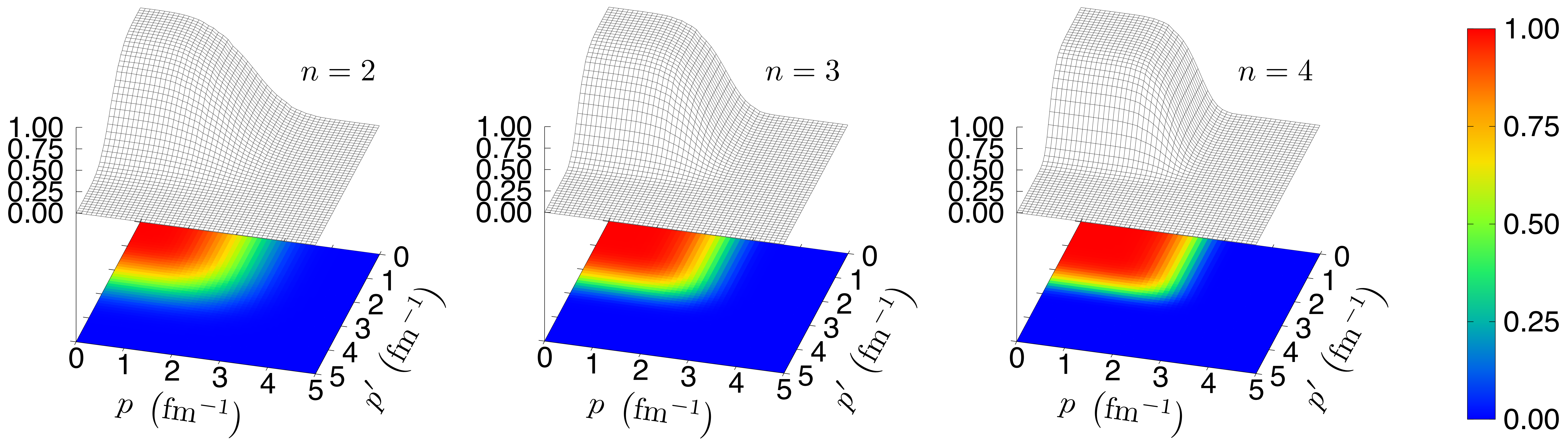
Nonlocal regularization

$$v_{2N}(\boldsymbol{p}', \boldsymbol{p}) \to v_{2N}(\boldsymbol{p}', \boldsymbol{p})\, u_n(p, p')$$

$$u_n(p, p') = \exp\left[-\left(\frac{p}{\Lambda_0}\right)^{2n} - \left(\frac{p'}{\Lambda_0}\right)^{2n}\right]$$

n	4 (One-pion exchange term at LO) 3 (Contact terms at LO) 2 (All other terms at NLO, N^2LO, and N^3LO)
Λ_0	500 MeV

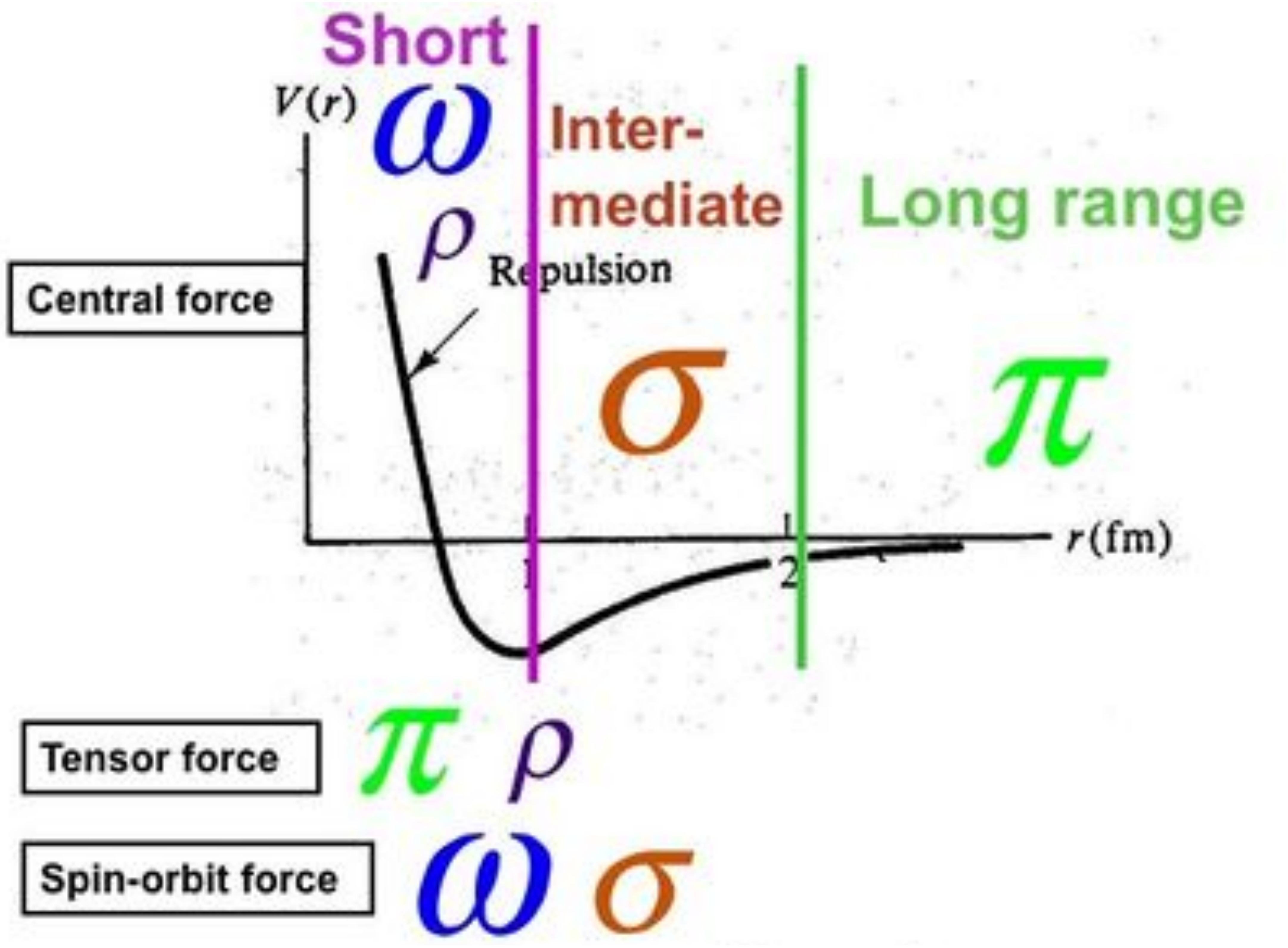
Machleidt & Entem, PR **503** 1 (2011)

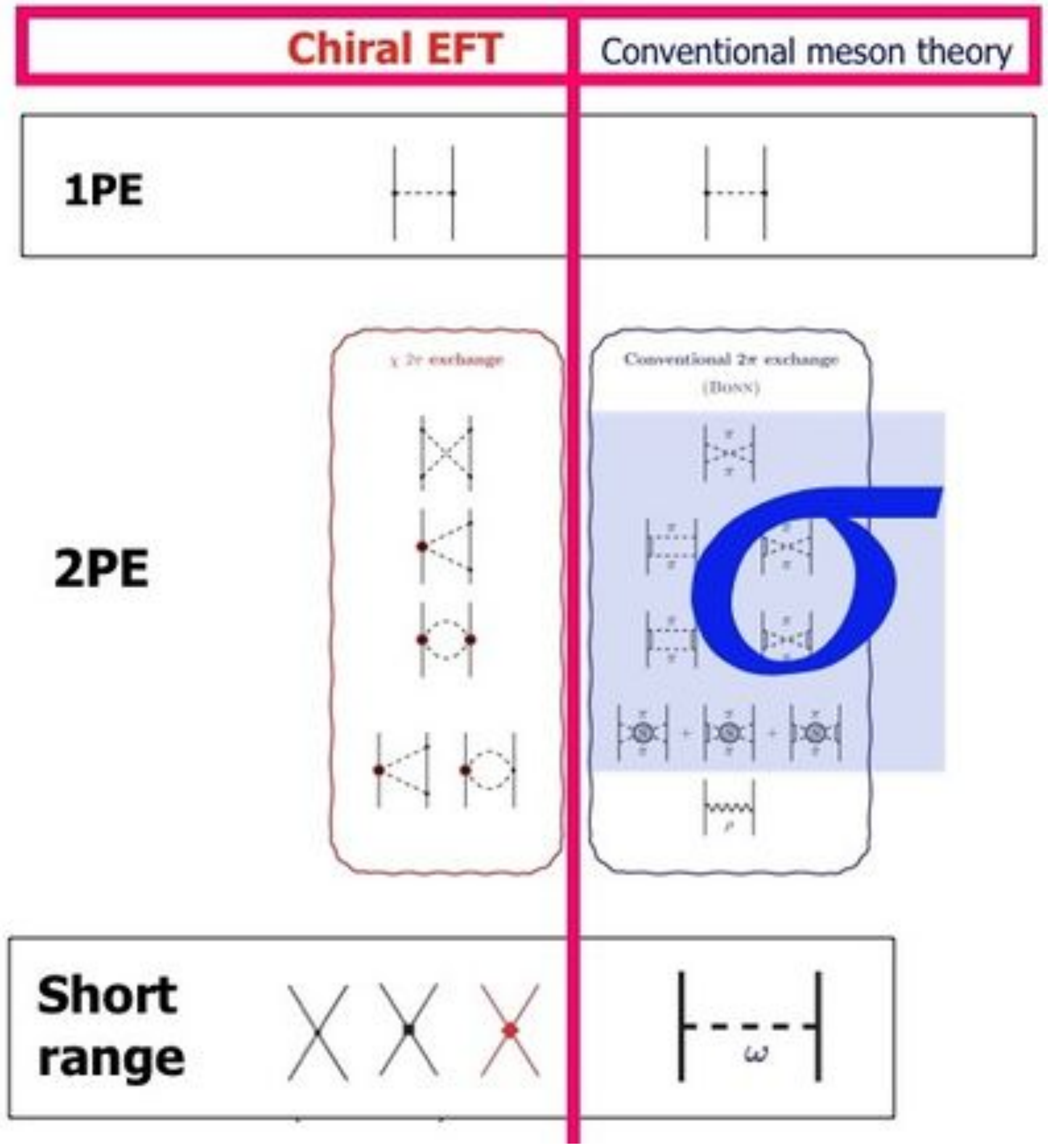


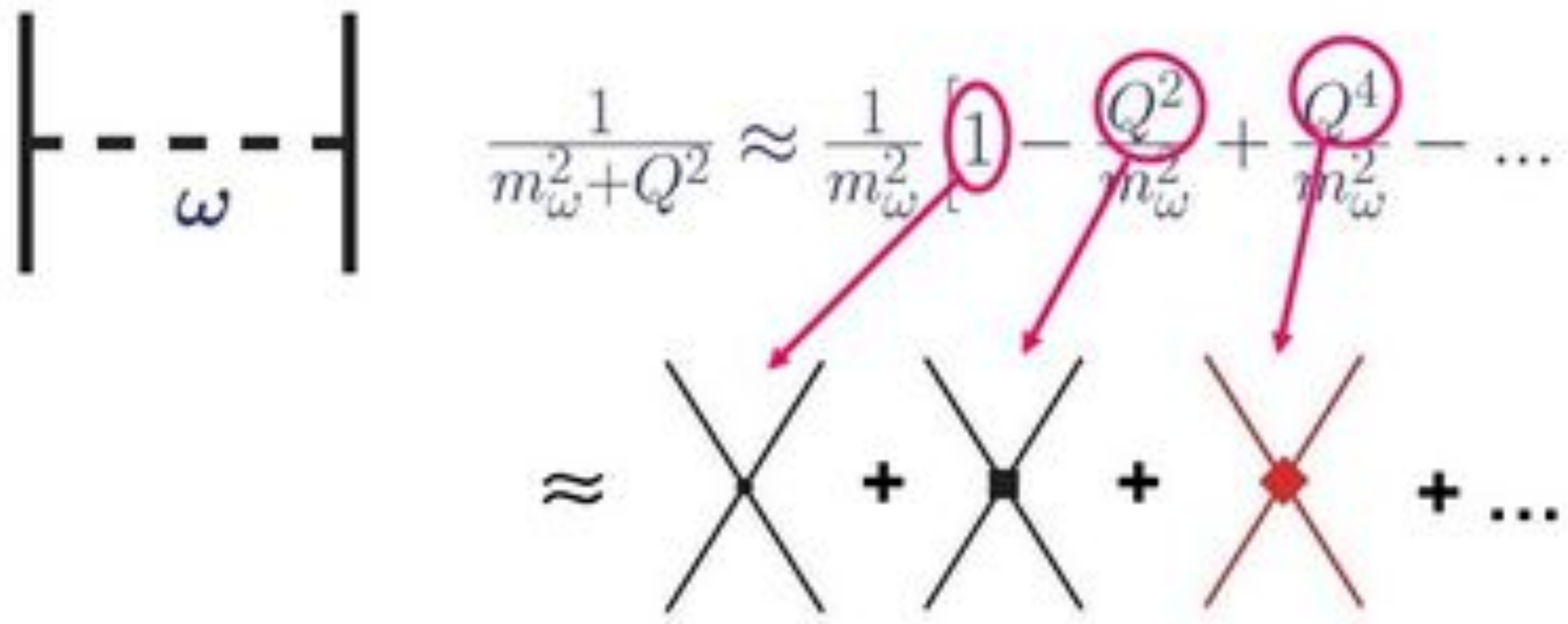
Chiral EFT and meson theory

Machleidt, PRC **63**, 024001 (2001)

scholarpedia.org/article/Nuclear_Forces







Problems of chiral EFT

Renormalization, power counting, and regularization

Weinberg power counting
with naive dimension analysis
→ Issues even at LO:

- Amplitudes and observables are not obtained
- Renormalization of 1π (nonperturbative)
- Lacks of contacts

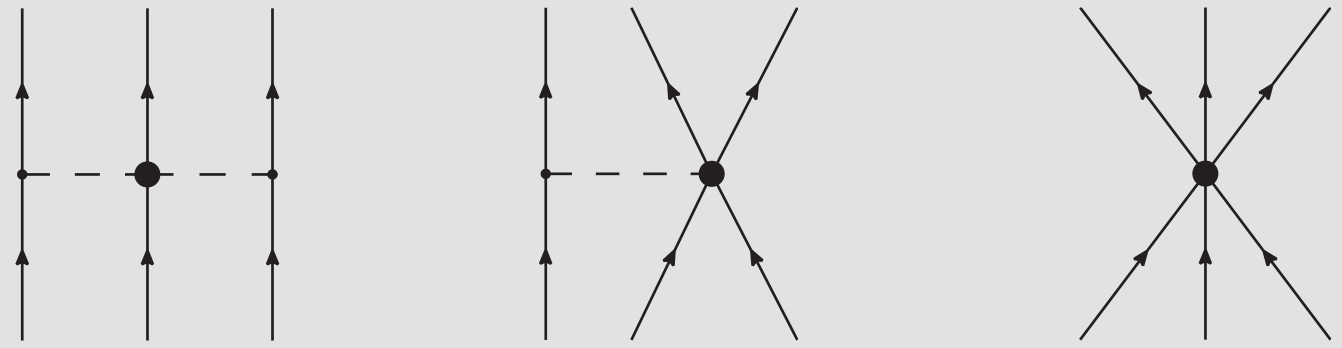
→ Alternative power counting and/or soft cutoff?

Intensive works ongoing!

Phenomenologically succeeded!

3NF LECs

Chiral N²LO 3NF



2π (c_1, c_3, c_4) | **1π + contact** (c_D) | **Contact** (c_E)

The LECs, c_D and c_E, cannot be fixed from 2N observables.

→ Their fitting procedures are not well settled.

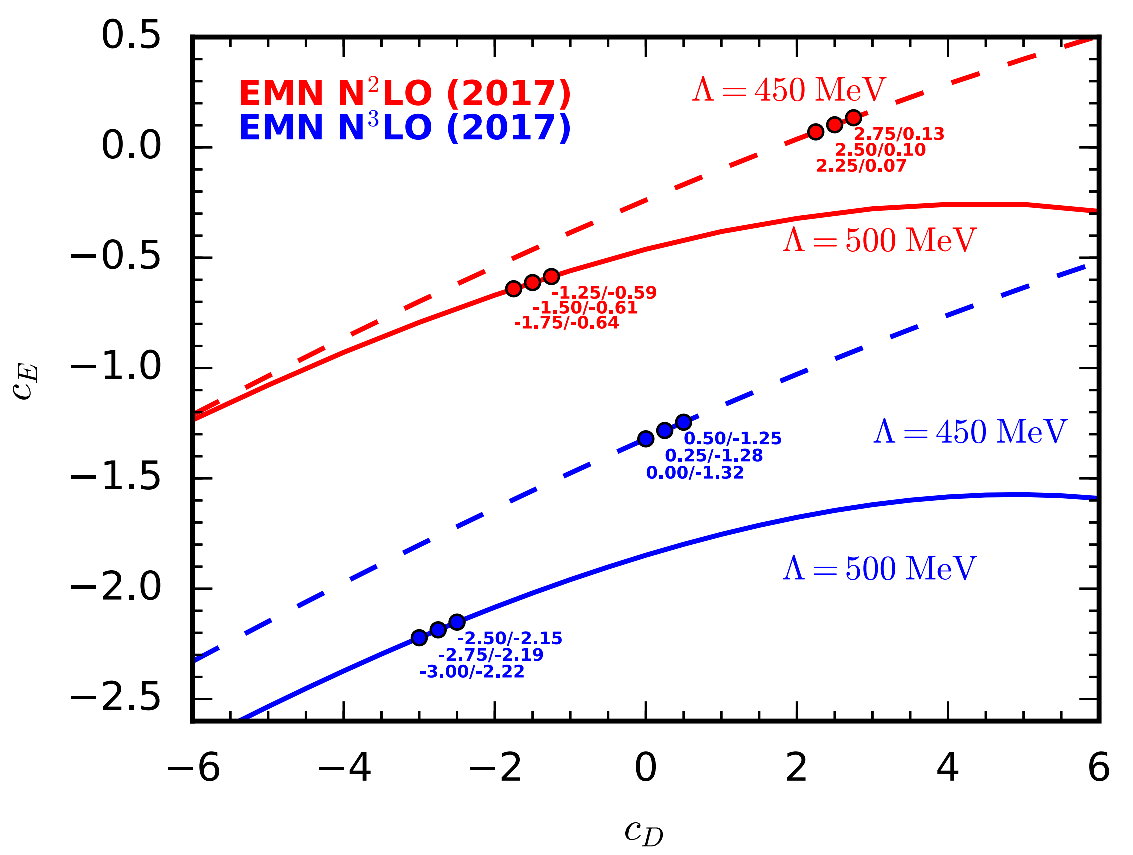
Comments on fitting procedures of 3N-LECs

Approach 1:
Fixed from purely 3-body systems

Phillips, NPA **107**, 209 (1968)

I personally believe this approach is appropriate.

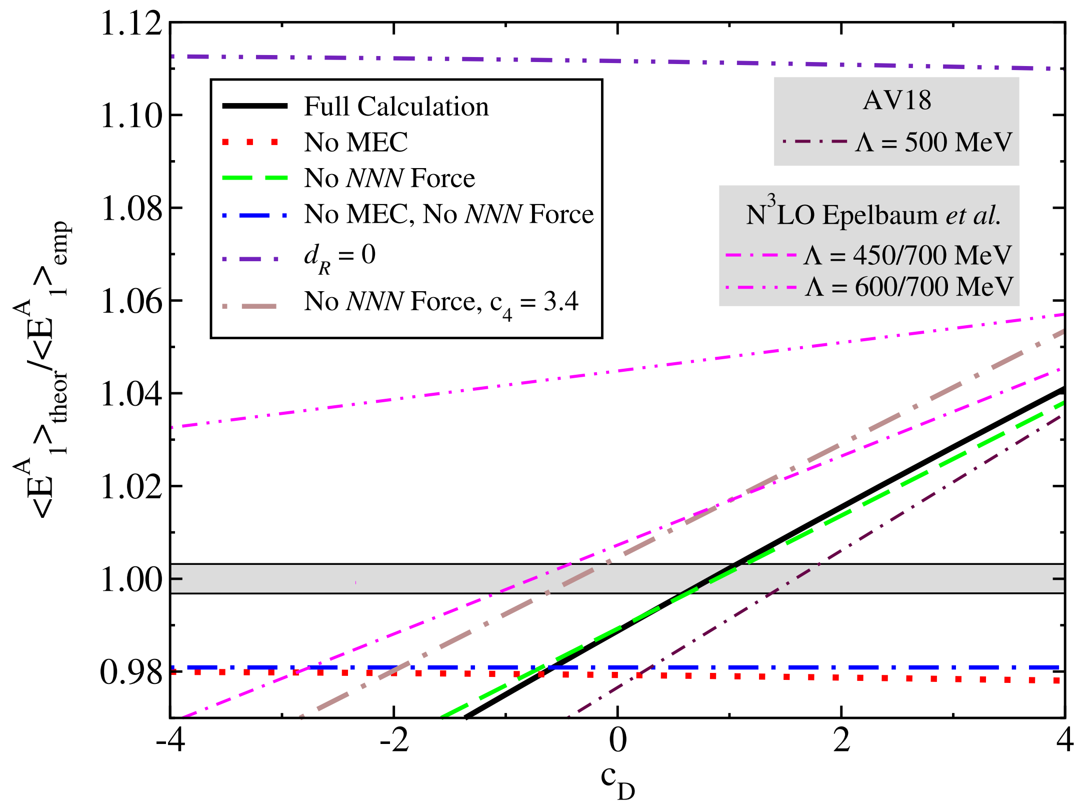
^{3}H/^{3}He g.s. energy



Hebeler, PR **890**, 1 (2021)

+

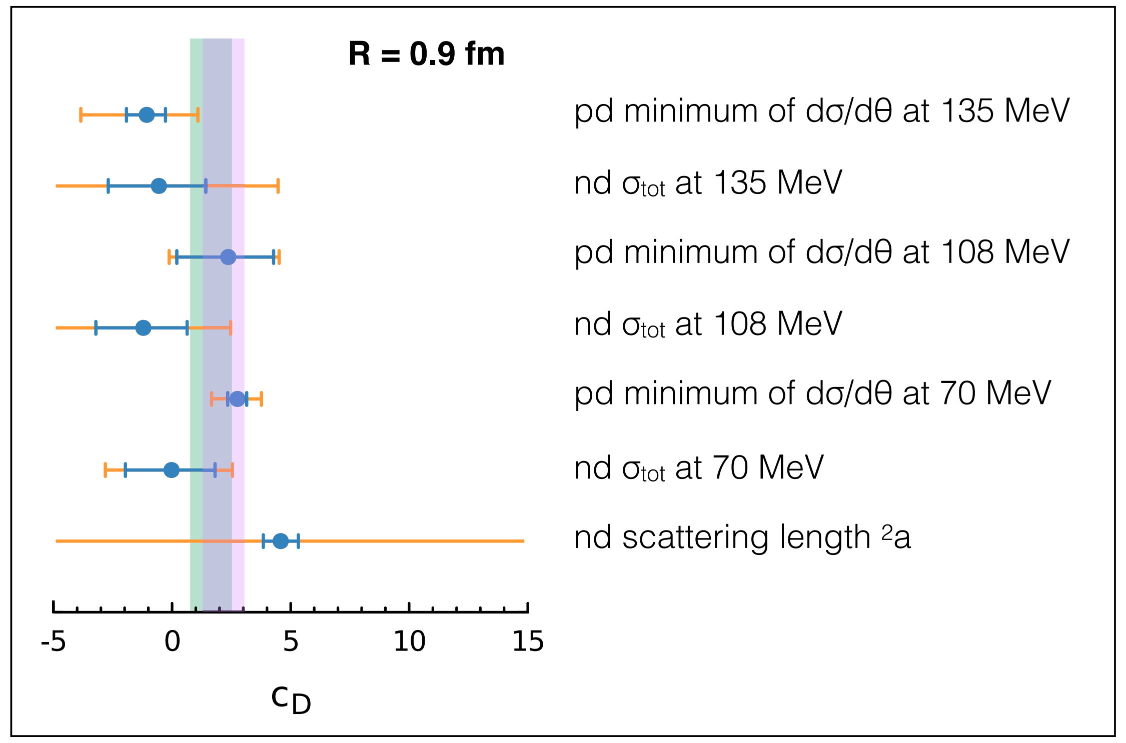
^{3}H-Gamow-Teller trans.



Gazit +, PRL **122**, 029901(E) (2019)

OR

N-*d* scattering



Epelbaum +, PRC **99**, 024313 (2019)

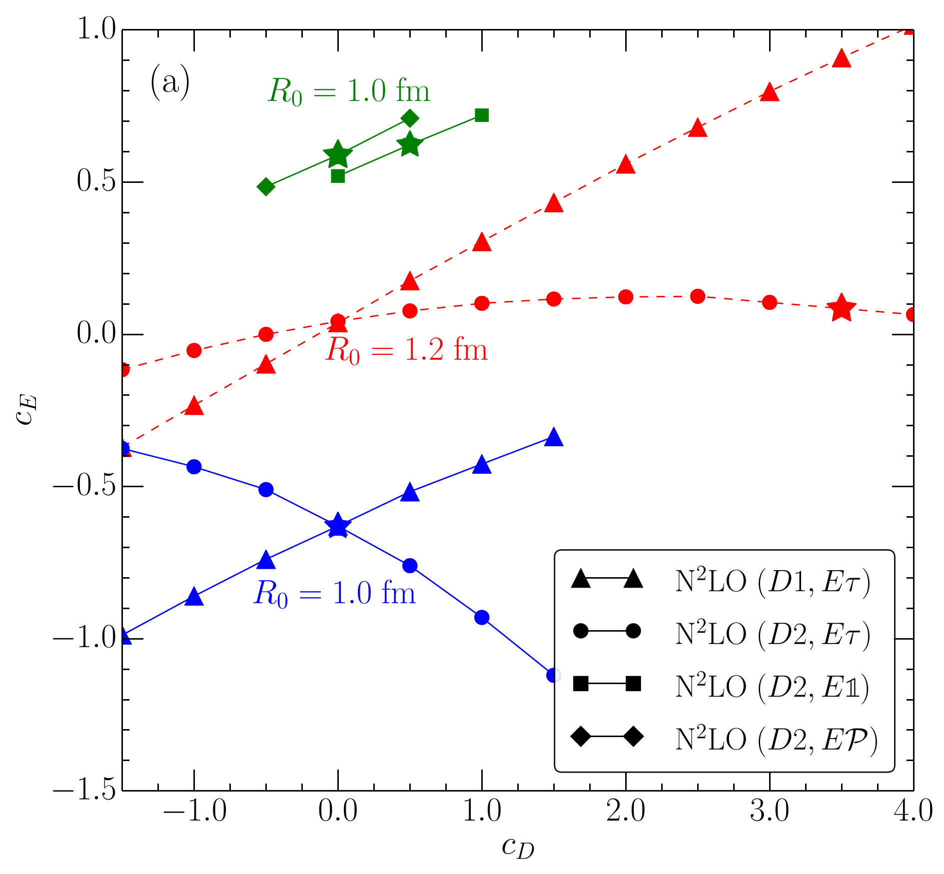
Review
Hebeler, PR **890**, 1 (2021)

Comments on fitting procedures of 3N-LECs

Approach 2:
Fixed from ^{4}He and/or heavier systems

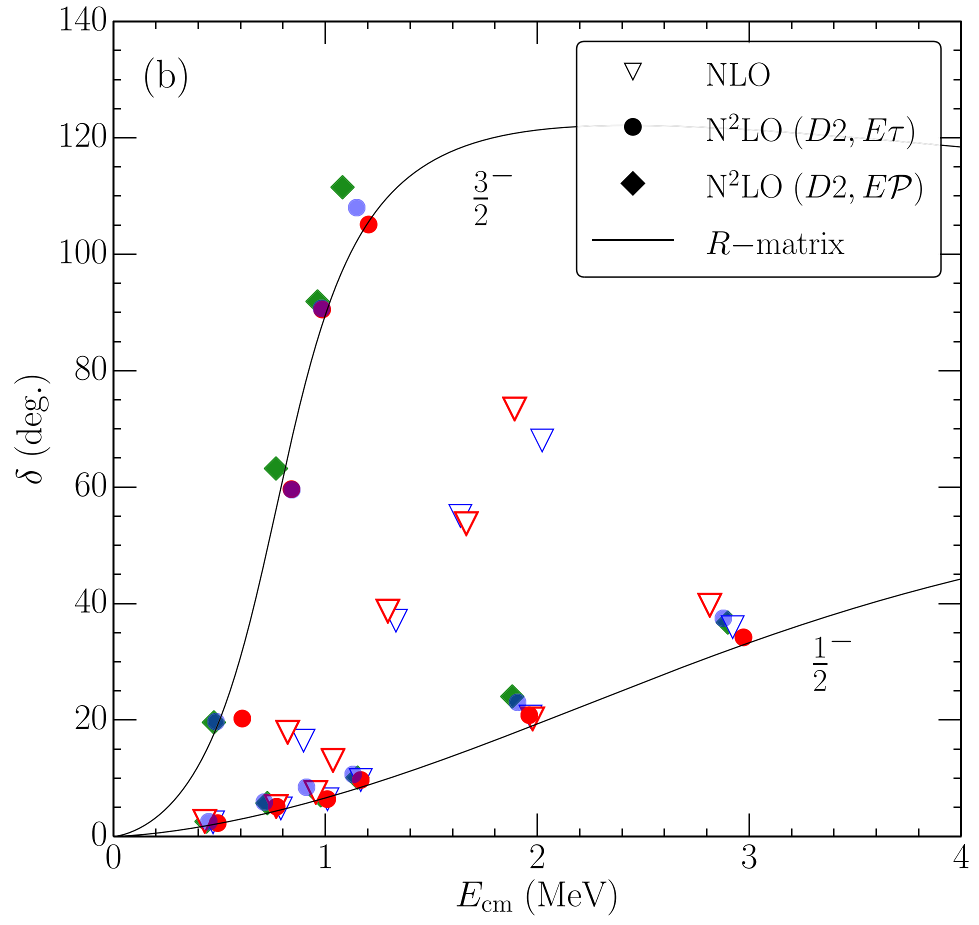
Tjon, PLB **56**, 217 (1975)

^{4}He g.s. energy



+

n-α scattering



Lynn, PRL **116**, 062501 (2016)

Review
Hebeler, PR ***890***, 1 (2021)

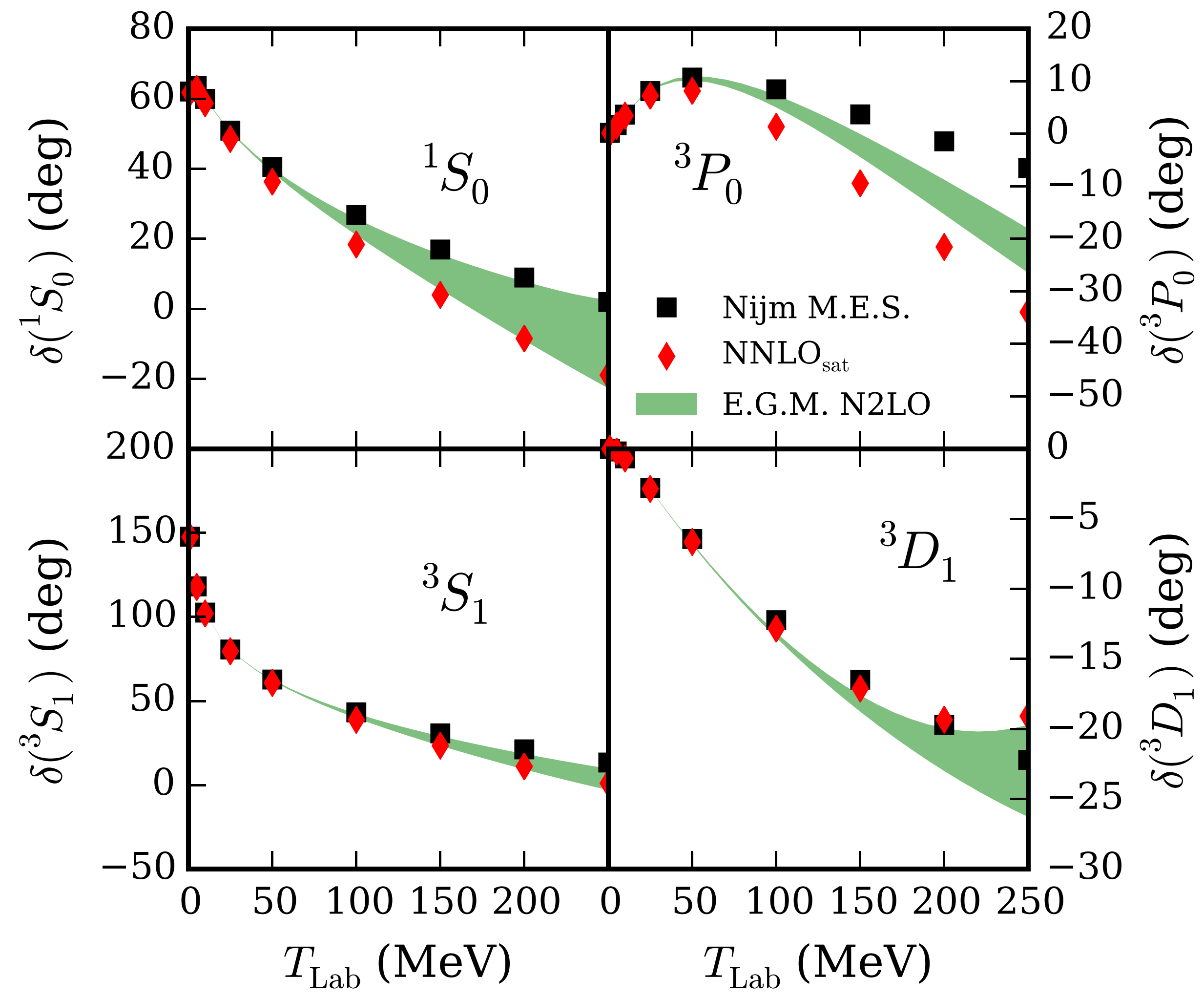
Comments on fitting procedures of 3N-LECs

Approach 3:
Totally phenomenological (NNLO$_{sat}$)

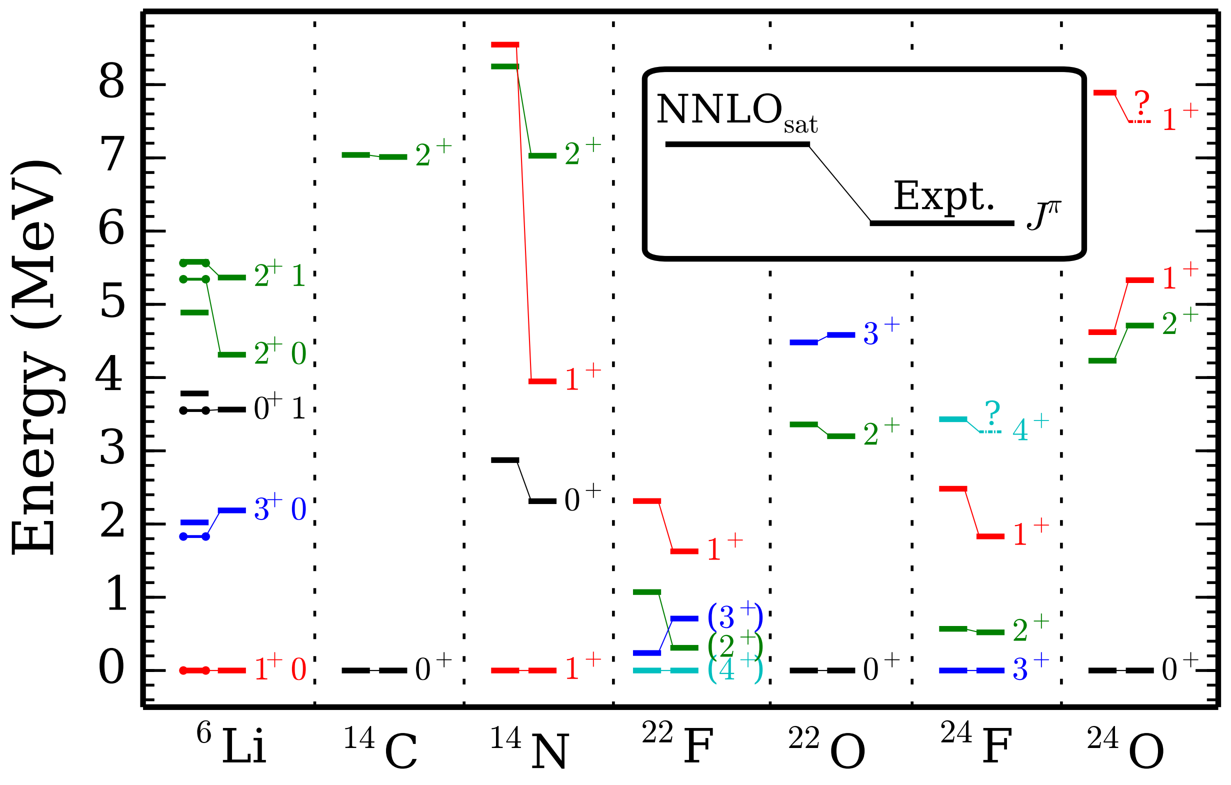
Ekström, **91**, 051301(R) (2015)

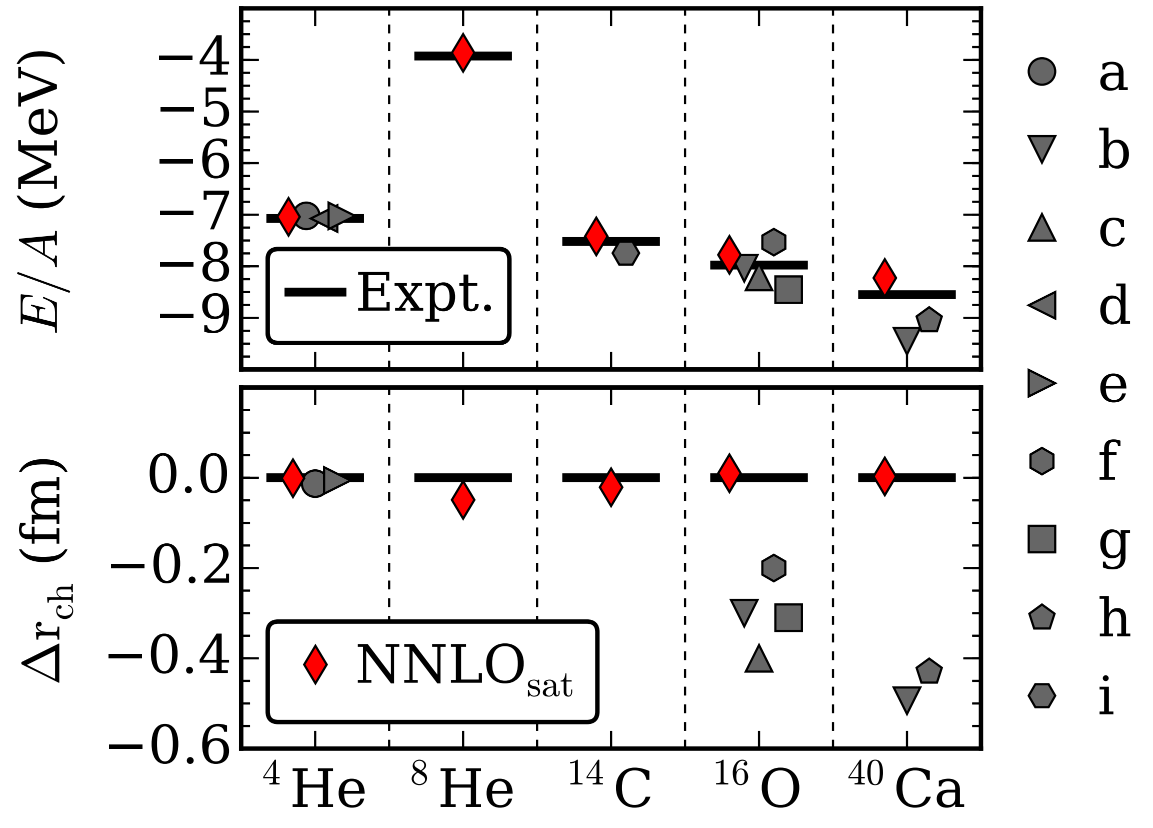
Is it a realistic force?

From few-body systems to carbon and oxygen isotopes



	NNLO$_{sat}$	N^{3}LO$_{EM}$ [49]	Expt.	Ref.
a^C_{pp}	−7.8258	−7.8188	−7.8196(26)	[70]
r^C_{pp}	2.855	2.795	2.790(14)	[70]
a_{nn}	−18.929	−18.900	−18.9(4)	[71]
r_{nn}	2.911	2.838	2.75(11)	[72]
a_{np}	−23.728	−23.732	−23.740(20)	[73]
r_{np}	2.798	2.725	2.77(5)	[73]
E_D	2.22457	2.22458	2.224566	[69]
r_D	1.978	1.975	1.97535(85)	[74]
Q_D	0.270	0.275	0.2859(3)	[73]

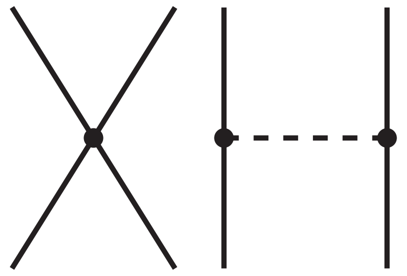
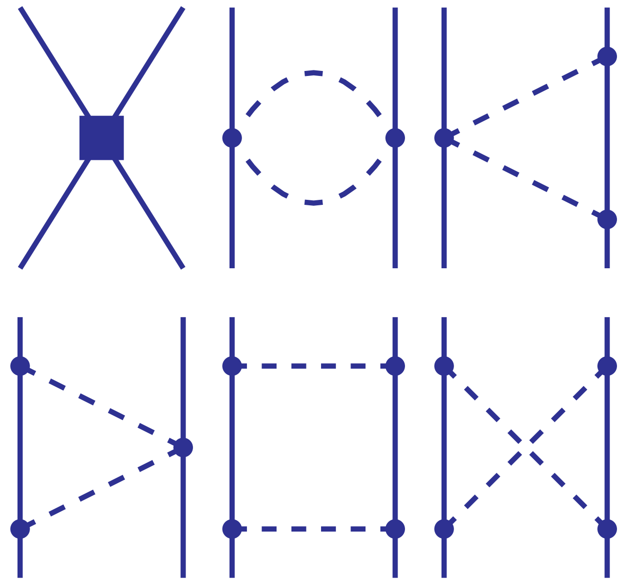
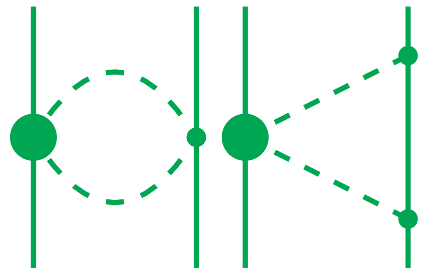
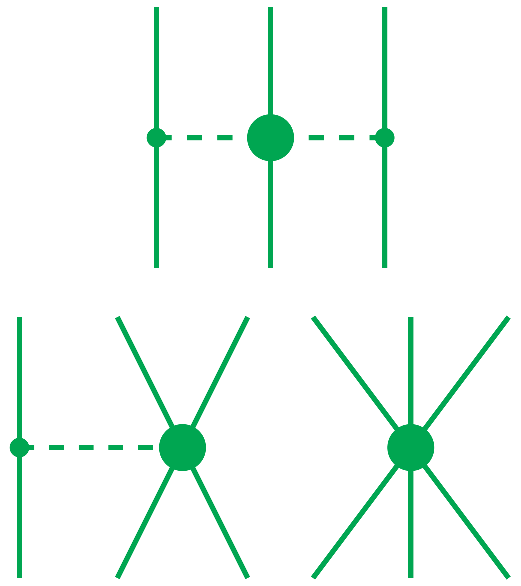




Chiral three-nucleon force

Hierarchy of chiral EFT

Entem +, PRC **96**, 024004 (2017)

	2N Force	3N Force
LO $(Q/\Lambda_\chi)^0$		
NLO $(Q/\Lambda_\chi)^2$		
NNLO $(Q/\Lambda_\chi)^3$		

Many-nucleon forces are derived consistently in chiral EFT

Hierarchy of chiral EFT

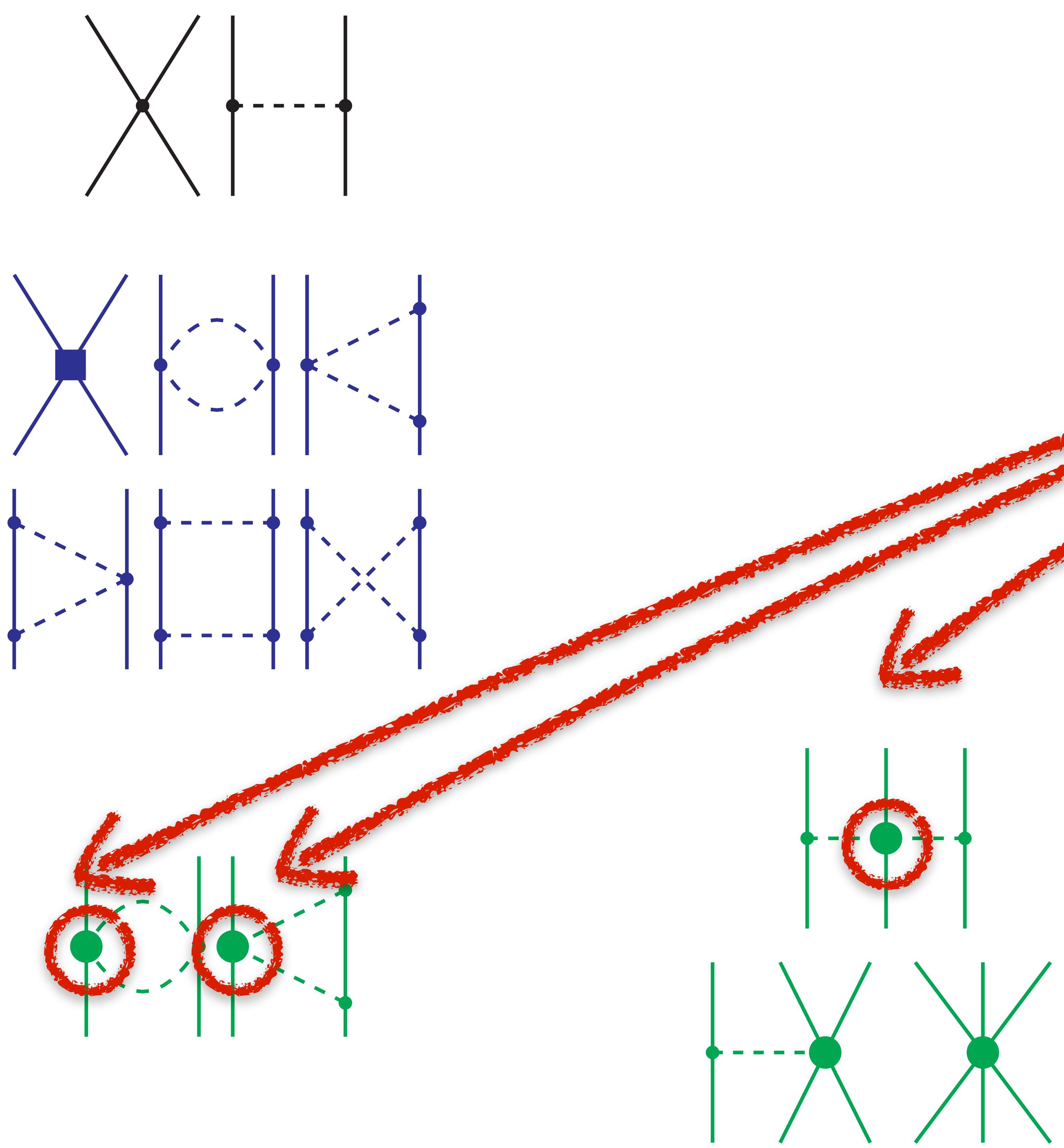
Entem +, PRC **96**, 024004 (2017)

2N Force **3N Force**

LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

NNLO
$(Q/\Lambda_\chi)^3$

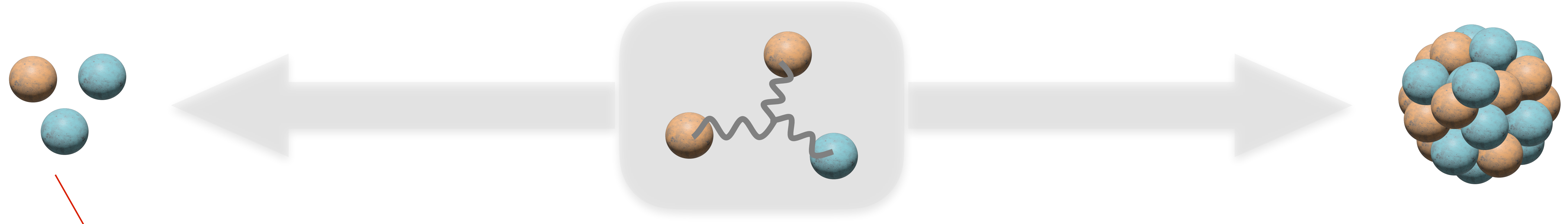


Many-nucleon forces are derived consistently in chiral EFT

Same coupling const.

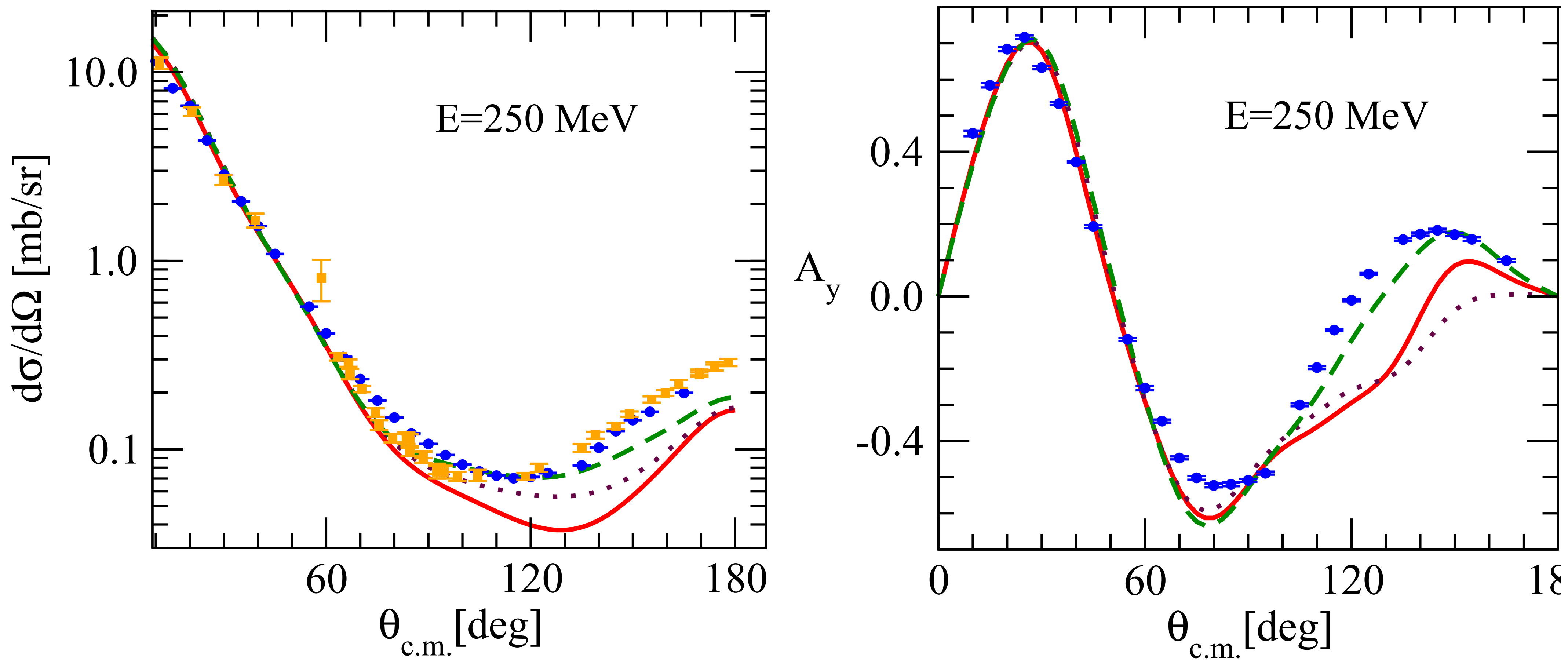
3NF | Few- and many-body systems

Few-body | 3NF | **Many-body** (my interest)



Fundamentals of 3NF

d + *p* scattering



Witała +, PRC **105**, 054004 (2022)

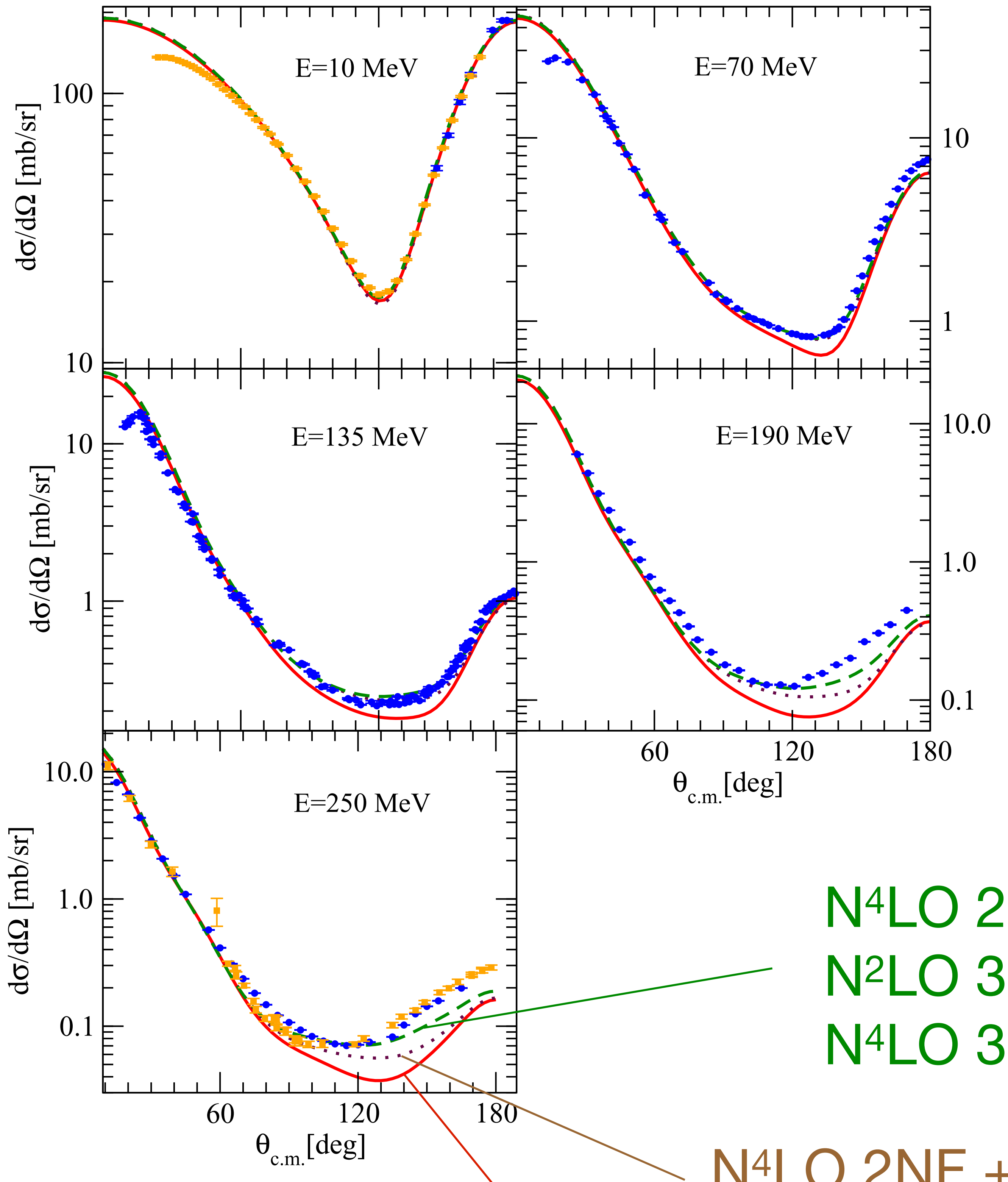
^{3}H/^{3}He structure

Interaction	Λ	$E(^3\mathrm{H})$	$E(^3\mathrm{He})$	ΔE_B
CD-Bonn+TM	4.784	−8.478	−7.735	0.743
AV18+TM	5.156	−8.478	−7.733	0.744
AV18+TM’	4.756	−8.448	−7.706	0.742
AV18+Urb-IX		−8.484	−7.739	0.745
AV18+Urb-IX (Pisa) [69]		−8.485	−7.742	0.743
AV18+Urb-IX (Argonne) [28]		−8.47(1)		
Expt.		−8.482	−7.718	0.764

Nogga +, PRC, **65**, 054003 (2002)

Sagara discrepancy

Witała +, PRC **105**, 054004 (2022)

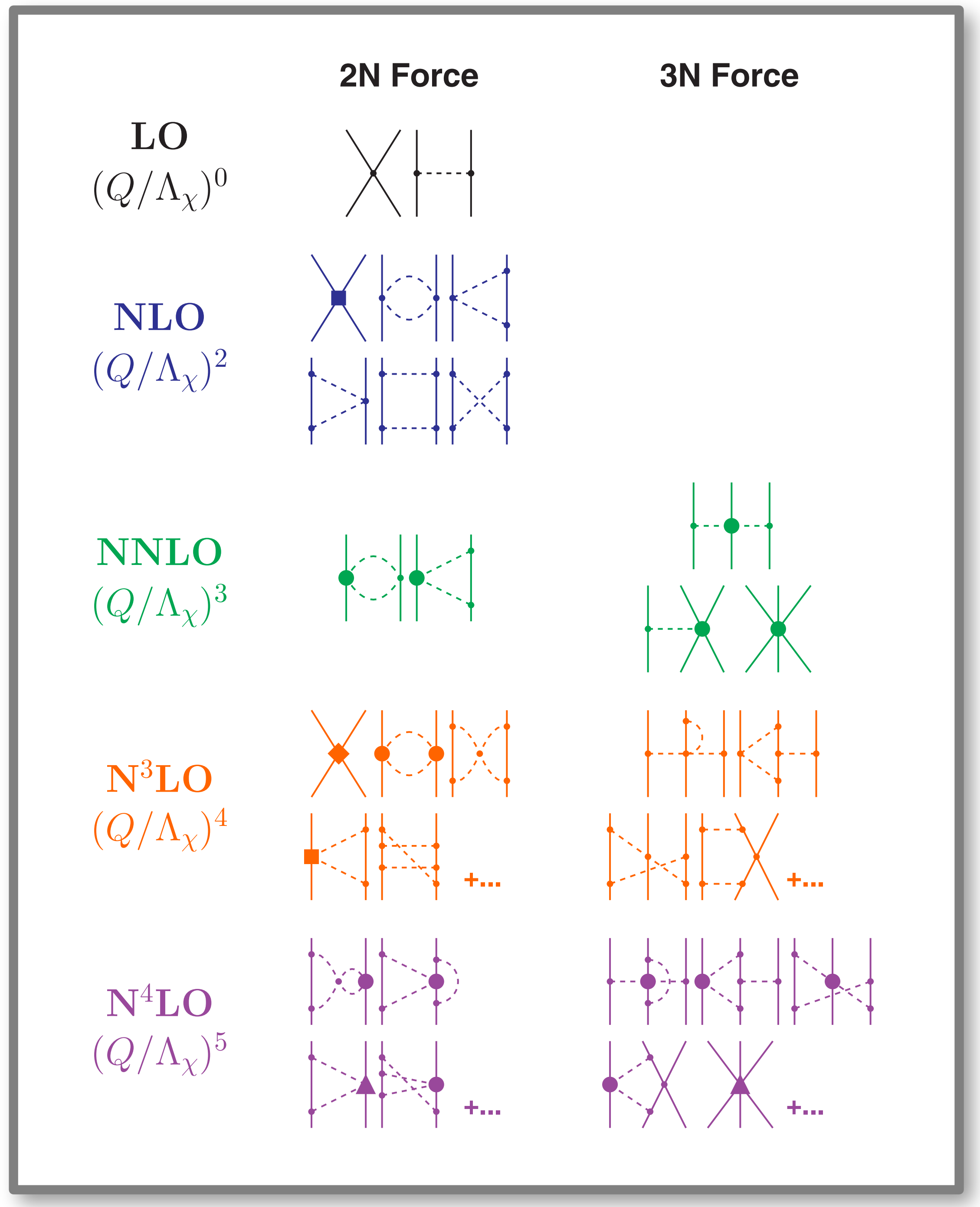


N⁴LO 2NF +
N²LO 3NF +
N⁴LO 3NF contacts

N⁴LO 2NF +
N²LO 3NF

N⁴LO 2NF

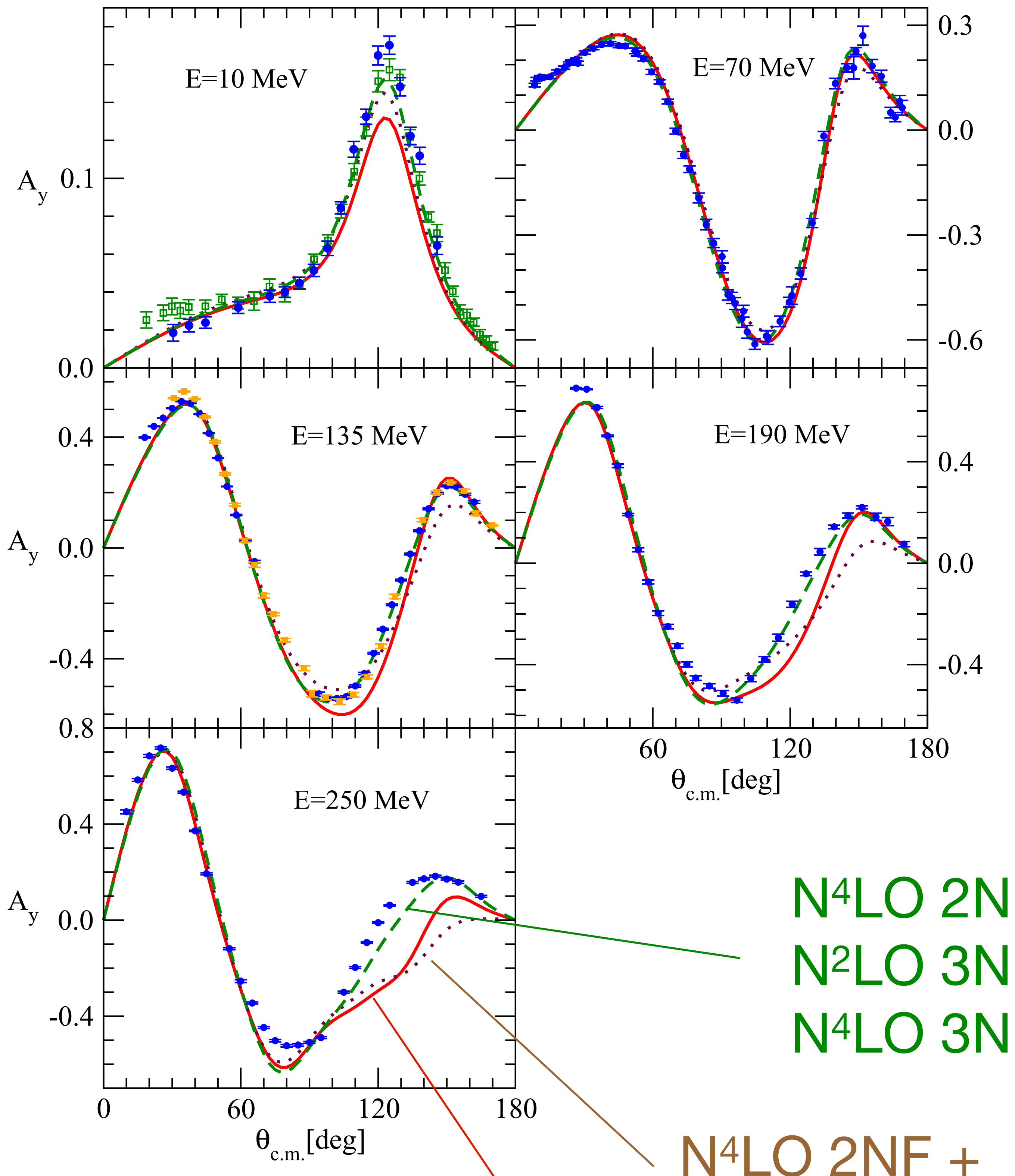
Entem +, PRC **96**, 024004 (2017)



Recent results | Chiral 3NF on few-nucleon systems

A_y puzzle

Witała +, PRC **105**, 054004 (2022)

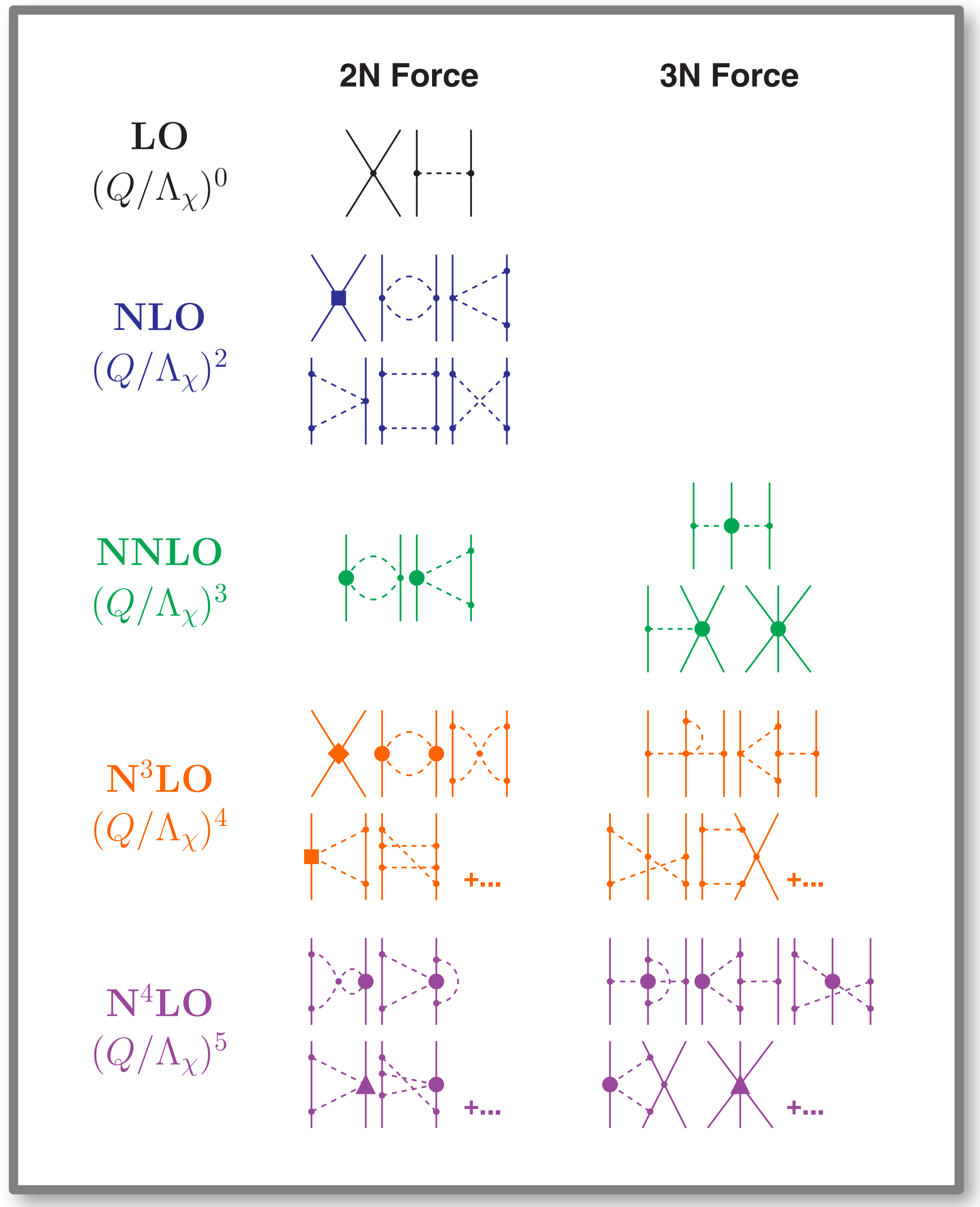


N⁴LO 2NF +
N²LO 3NF +
N⁴LO 3NF contacts

N⁴LO 2NF +
N²LO 3NF

N⁴LO 2NF

Entem +, PRC **96**, 024004 (2017)

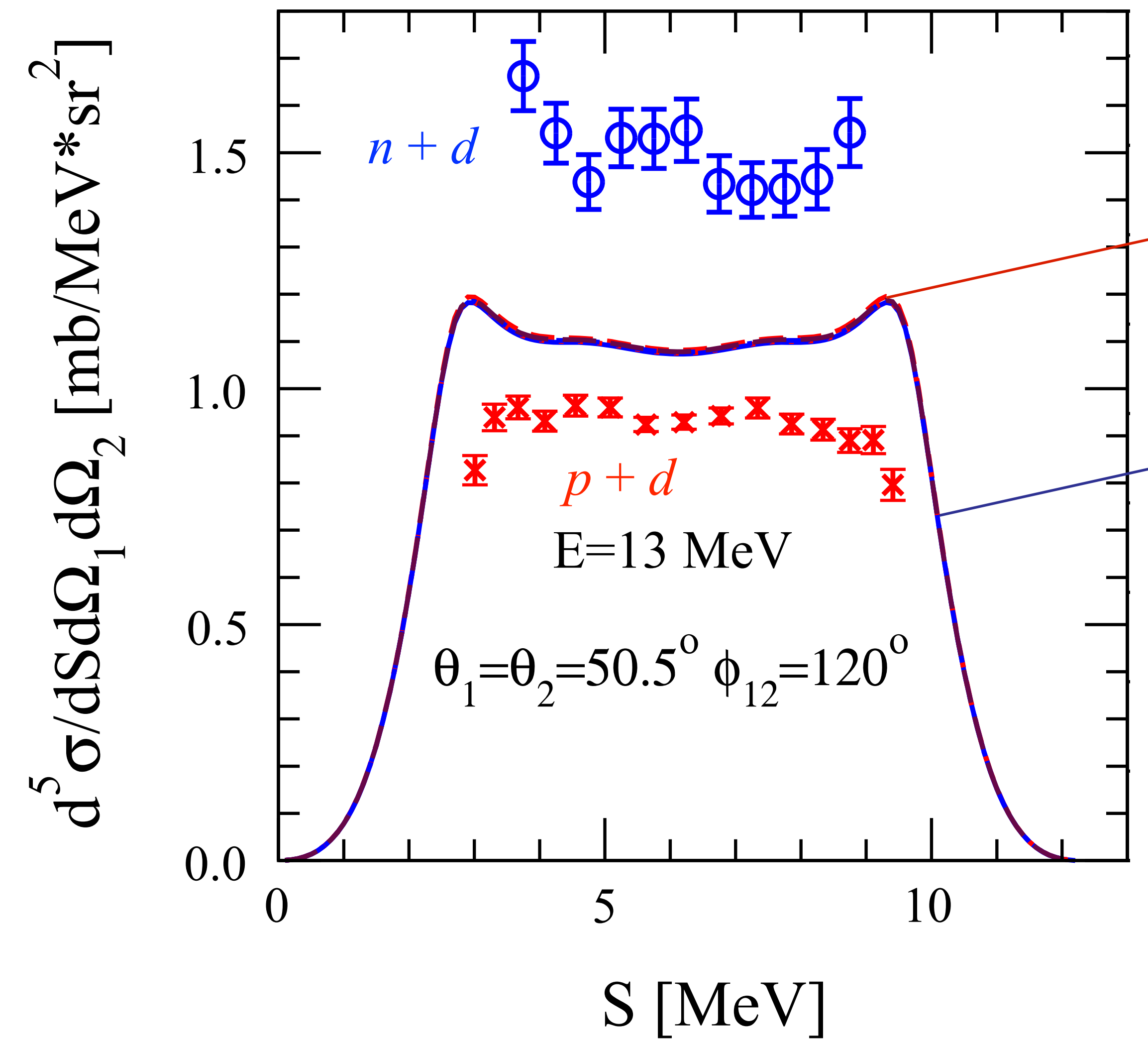


Cf. also Girlanda +, arXiv:2302.03468

Recent results | Chiral 3NF on few-nucleon systems

Space-star anomaly

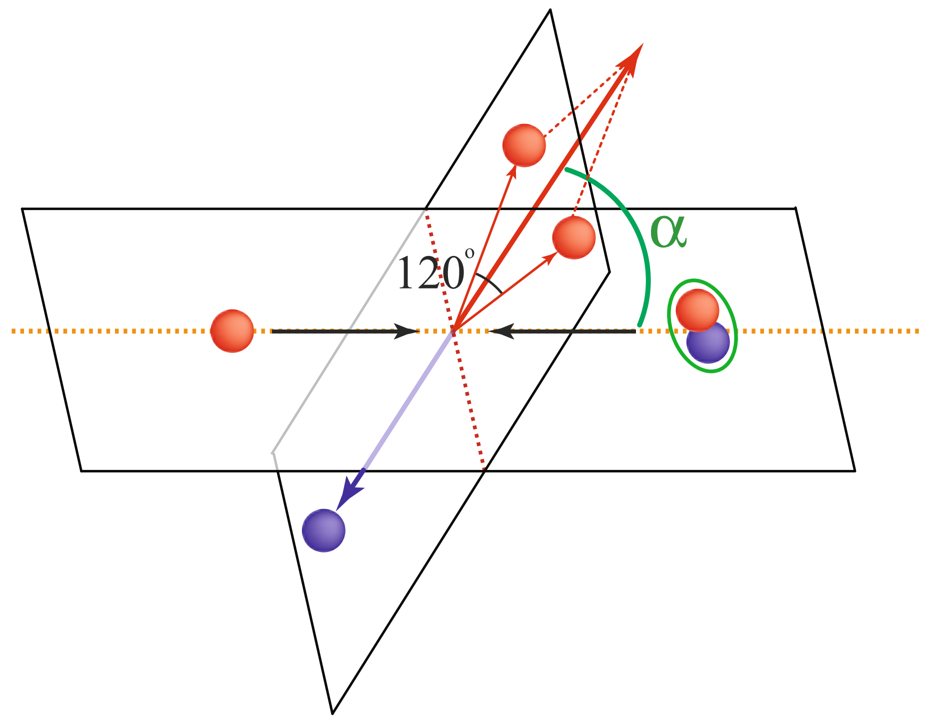
Witała +, Few-Body Syst **60**, 19 (2019)



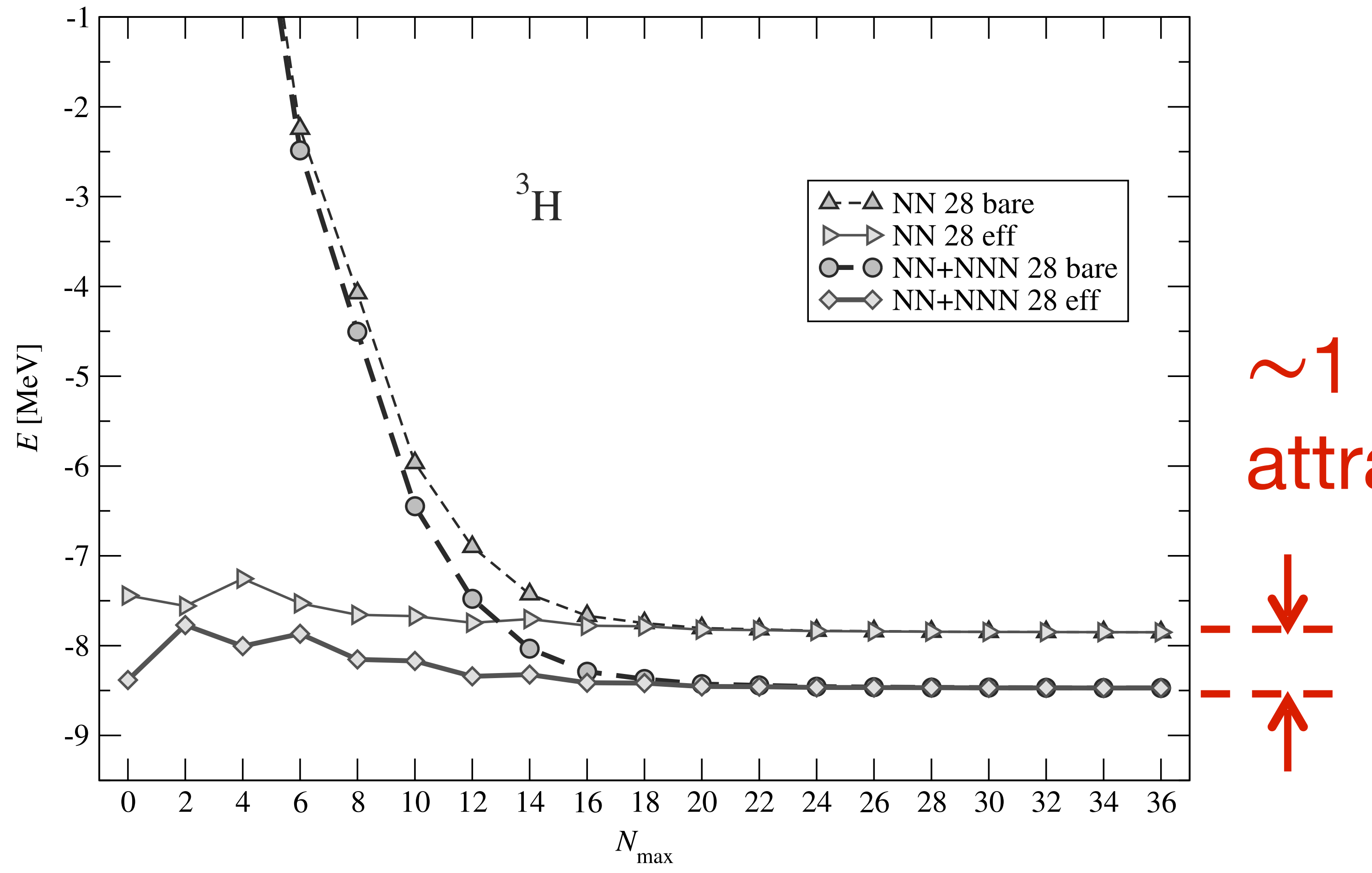
N⁴LO 2NF

N⁴LO 2NF + N²LO 3NF
(4 lines: Diff. param.)

Yabe +, Few-Body Syst **50**, 291 (2011)



Triton-biding energy



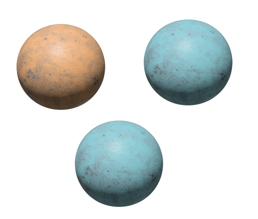
~1 MeV attraction

ab initio calc.

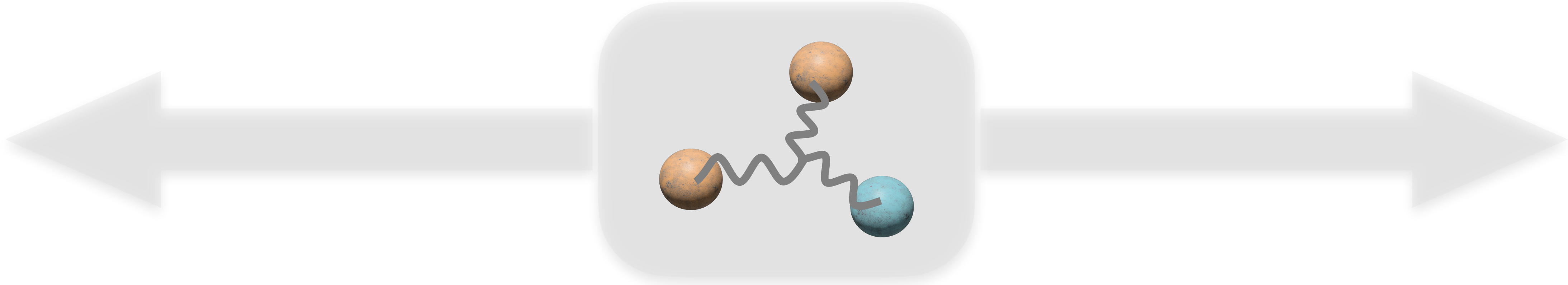
Navrátil, FBS **41**, 117 (2007)

3NF | Few- and many-body systems

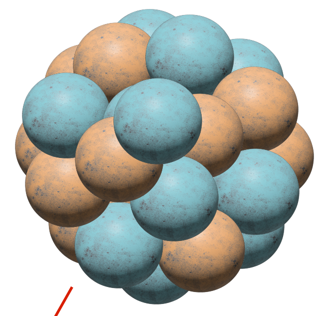
Few-body



3NF

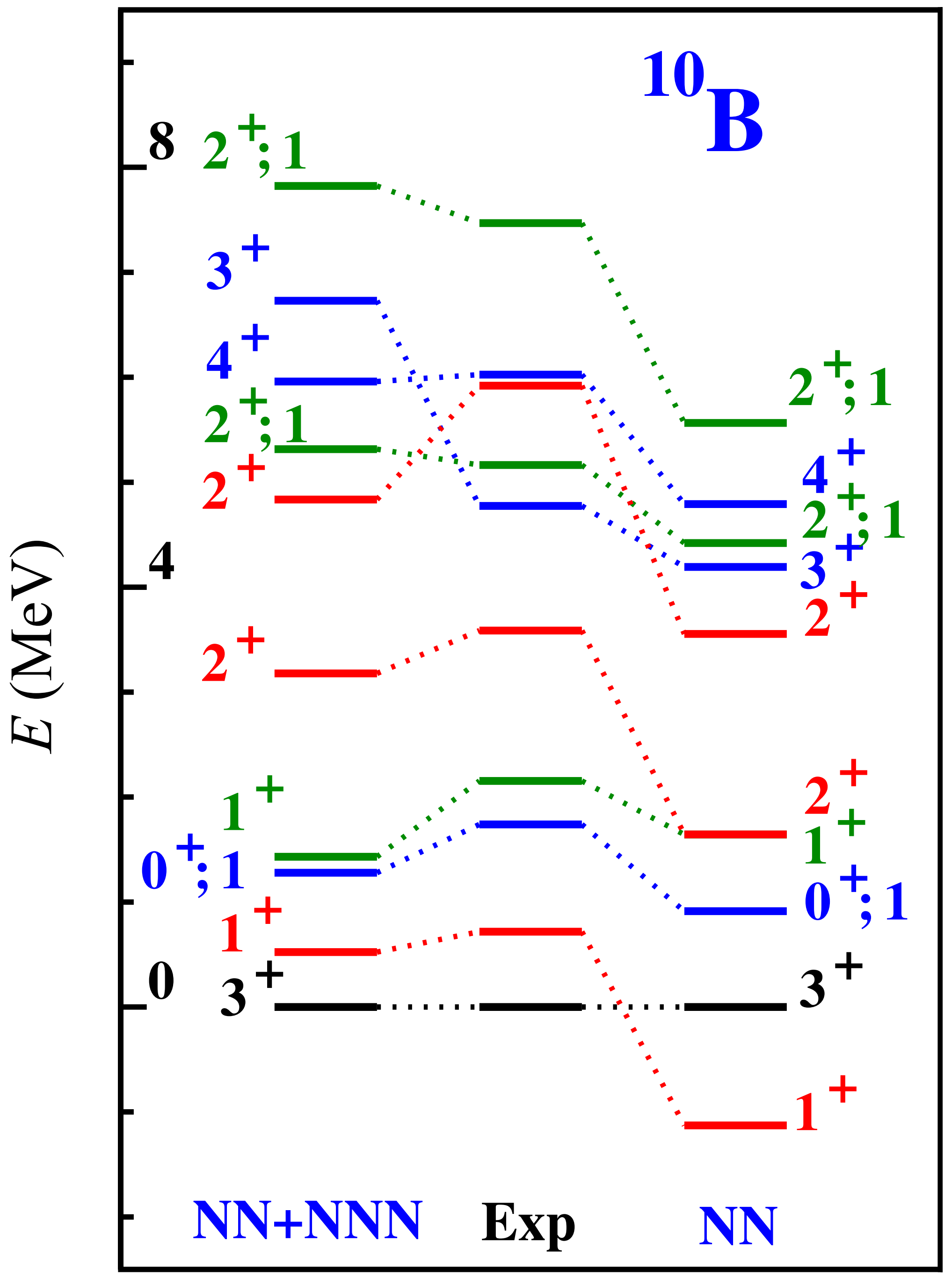


Many-body (my interest)



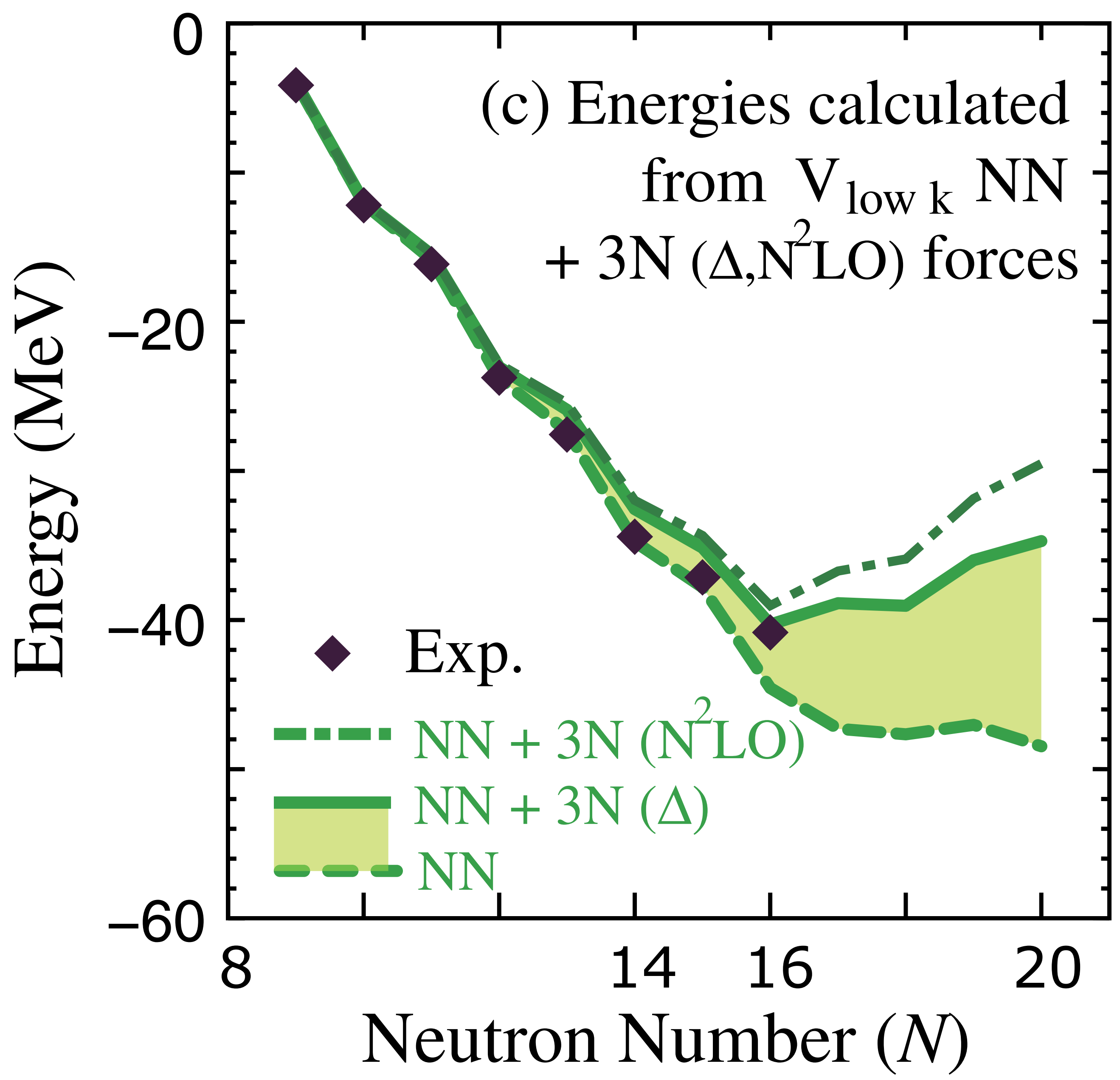
Major playground of 3NF

^{10}B spectra



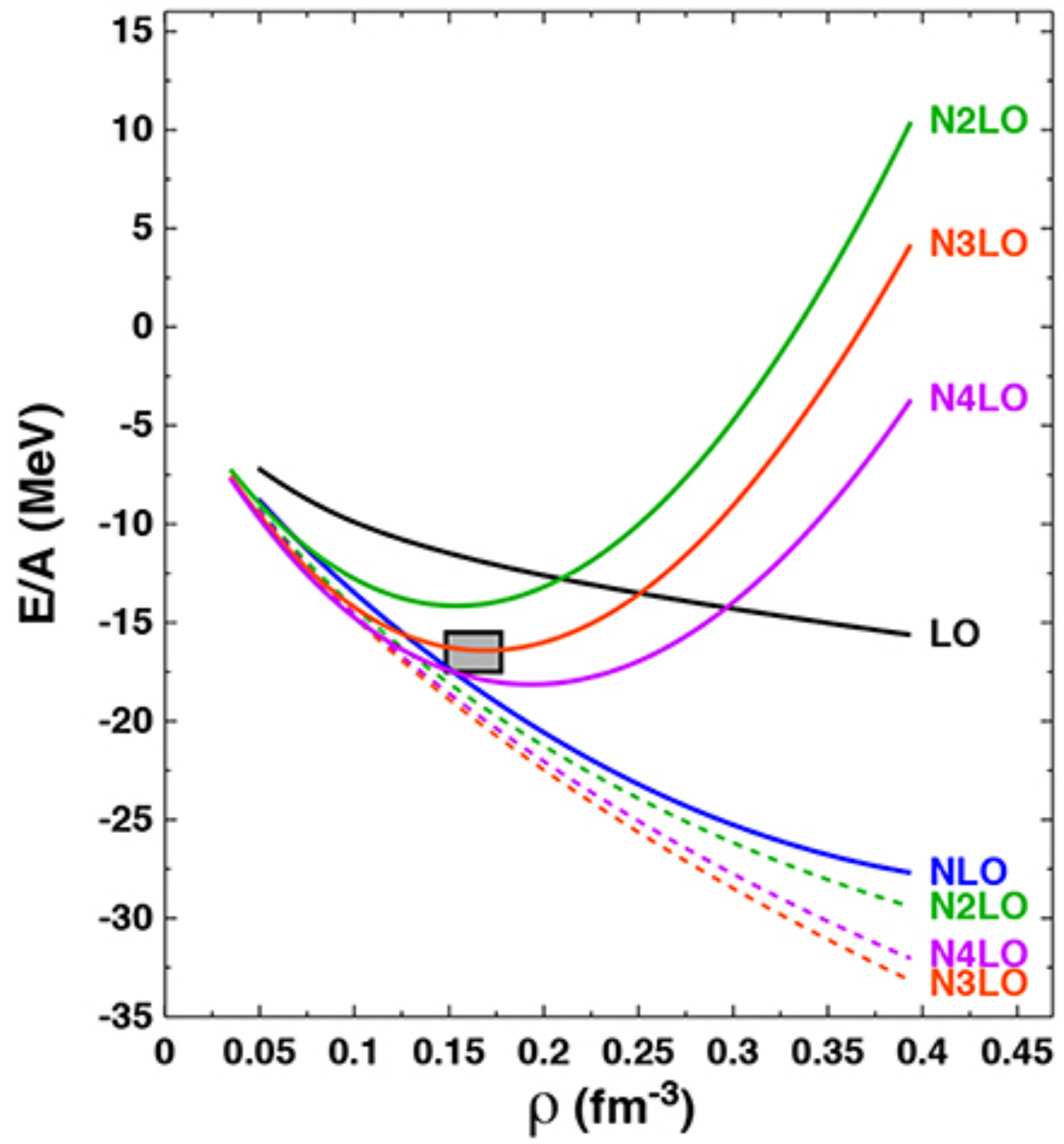
Navrátil +, PRL **99**, 042501 (2007)

Oxygen-drip line



Otsuka +, PRL **105**, 032501 (2010)

Nuclear matter saturation

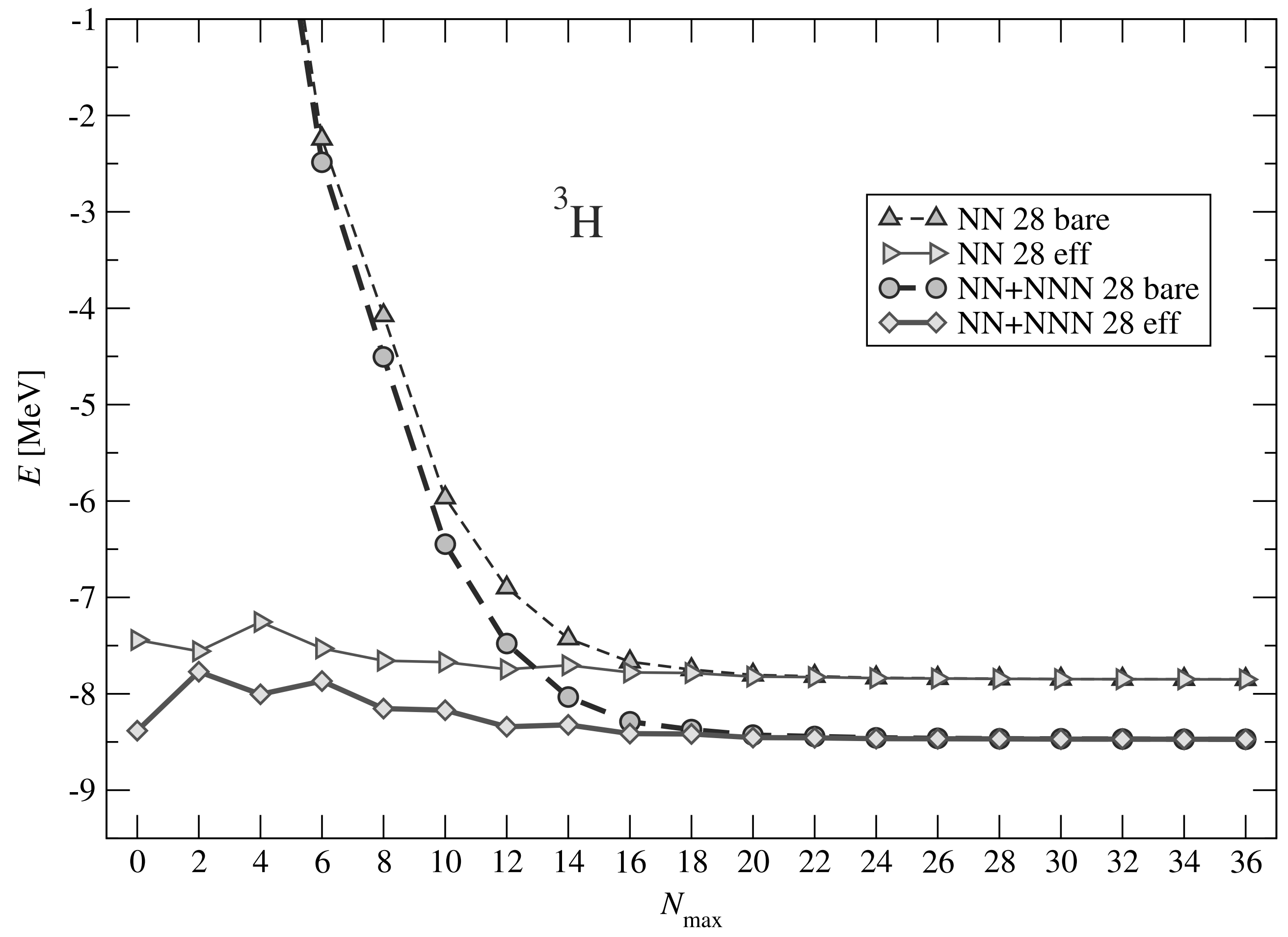


Sammarruca & Millerson, Front. Phys. **7**, 00213 (2019)

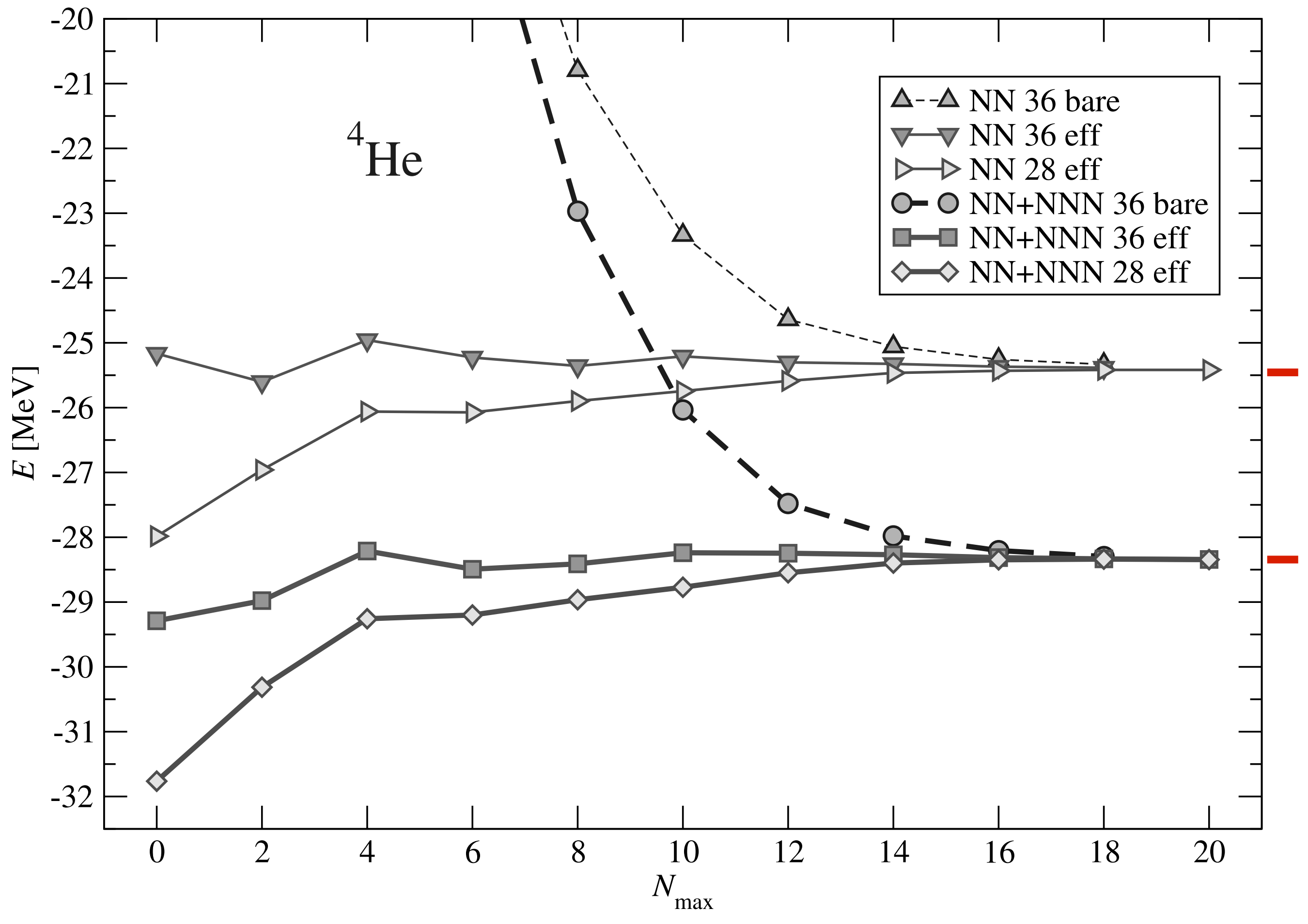
3NF contributions to ground-state energies

s-shell nuclei

NCSM Navrátil, FBS **41**, 117 (2007)



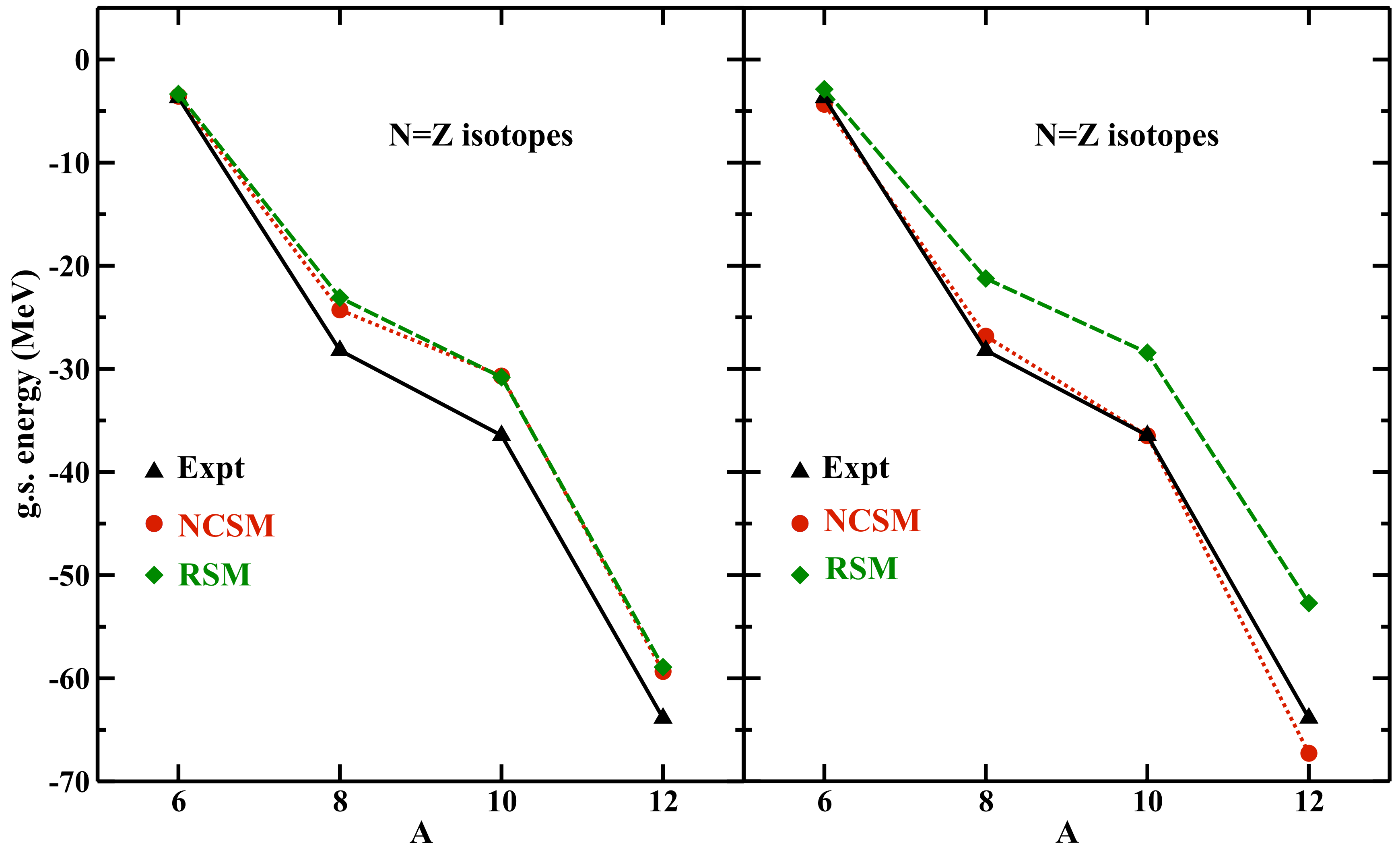
~1 MeV attraction



~3 MeV attraction

p-shell nuclei

2NF only 2NF + 3NF



NCSM

Navrátil +, PRL **99**, 042501 (2007)

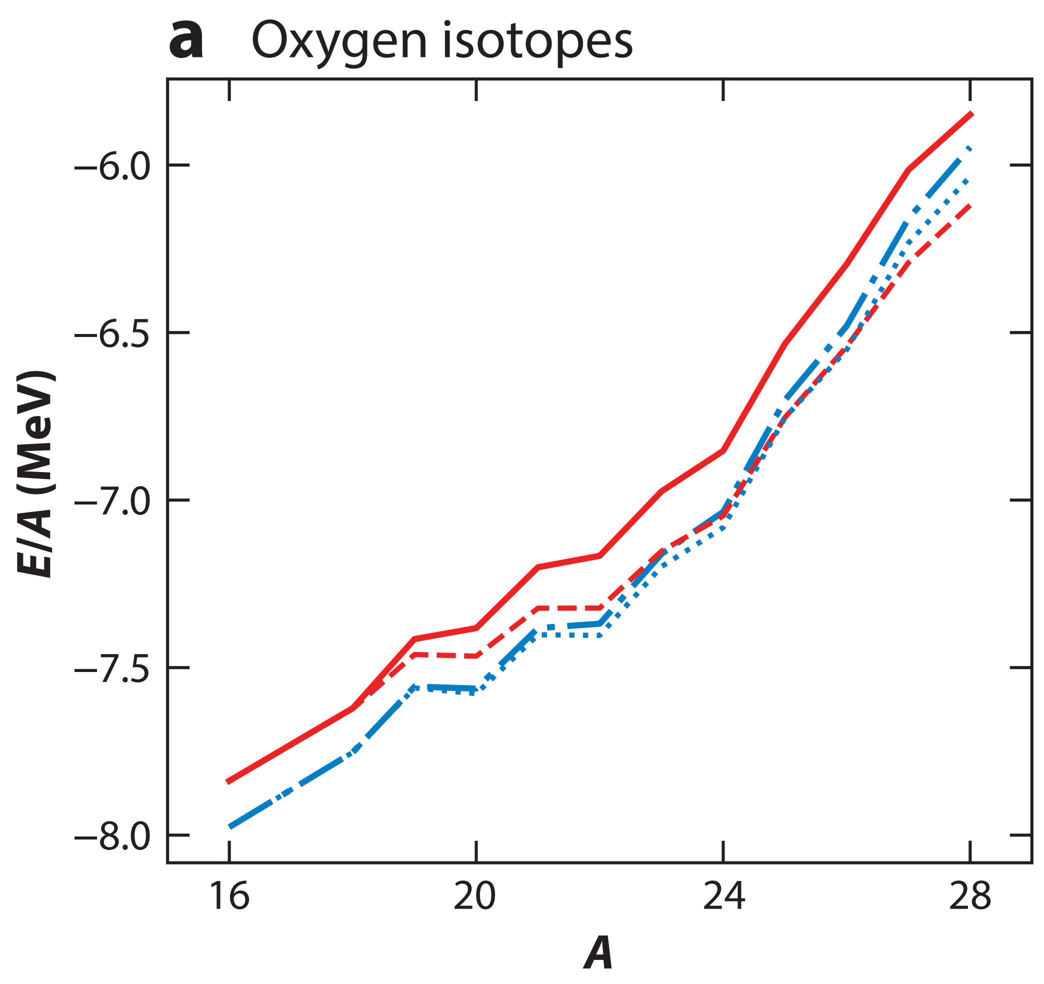
~ 5—10 MeV attraction

3NF contributions to ground-state energies

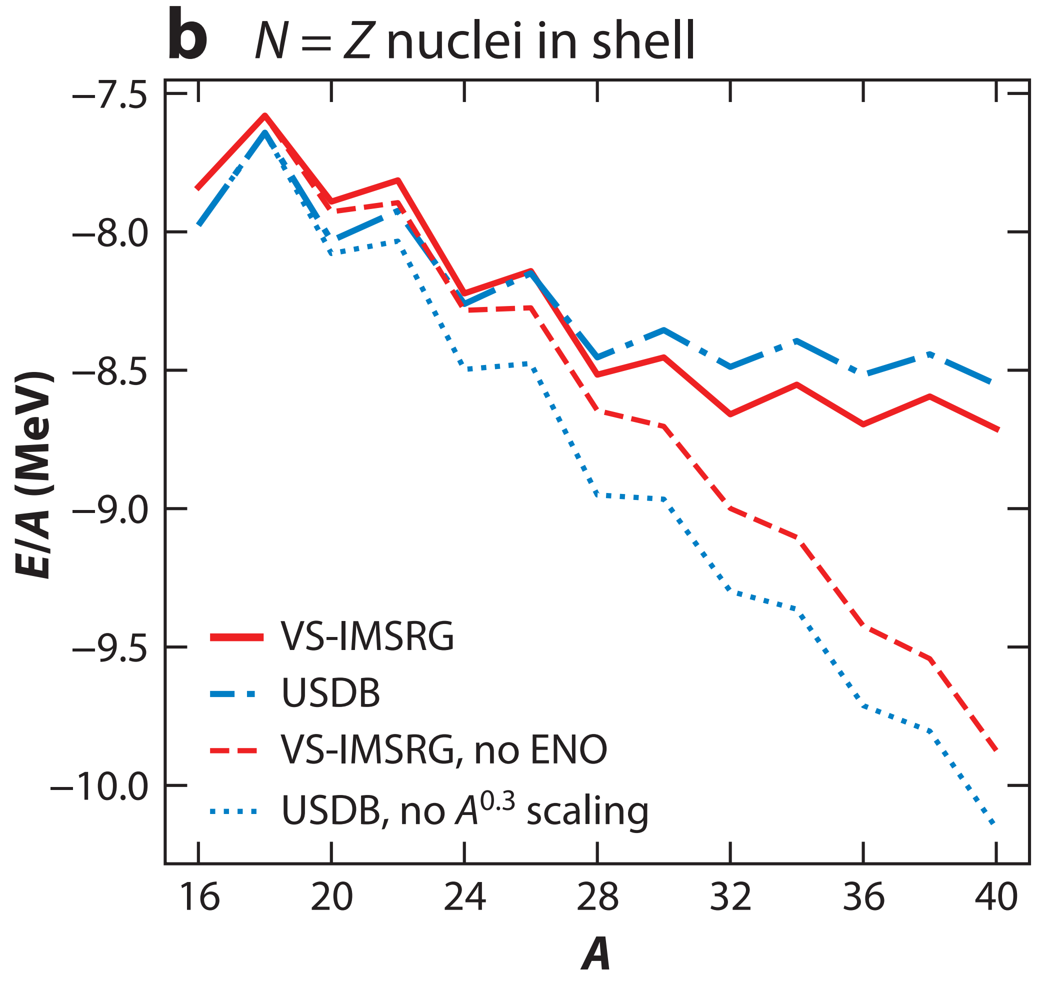
sd-shell nuclei

Valence-space in-medium SRG

Stroberg+, ARNPS **69**, 307 (2019)

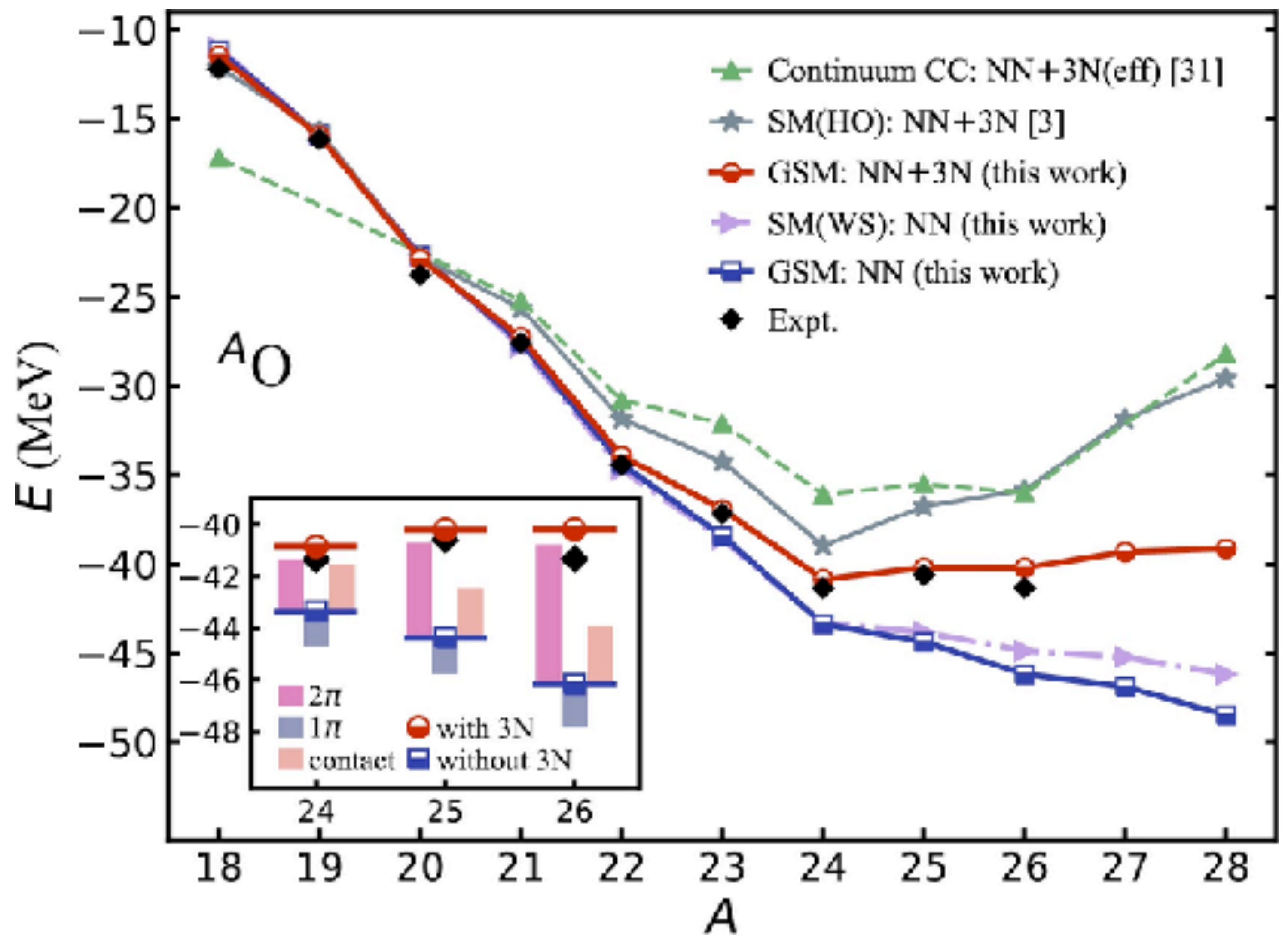


≲ 0.3 MeV/A repulsion



≲ 1.5 MeV/A repulsion

Oxygen (*sd*-shell + continuum)



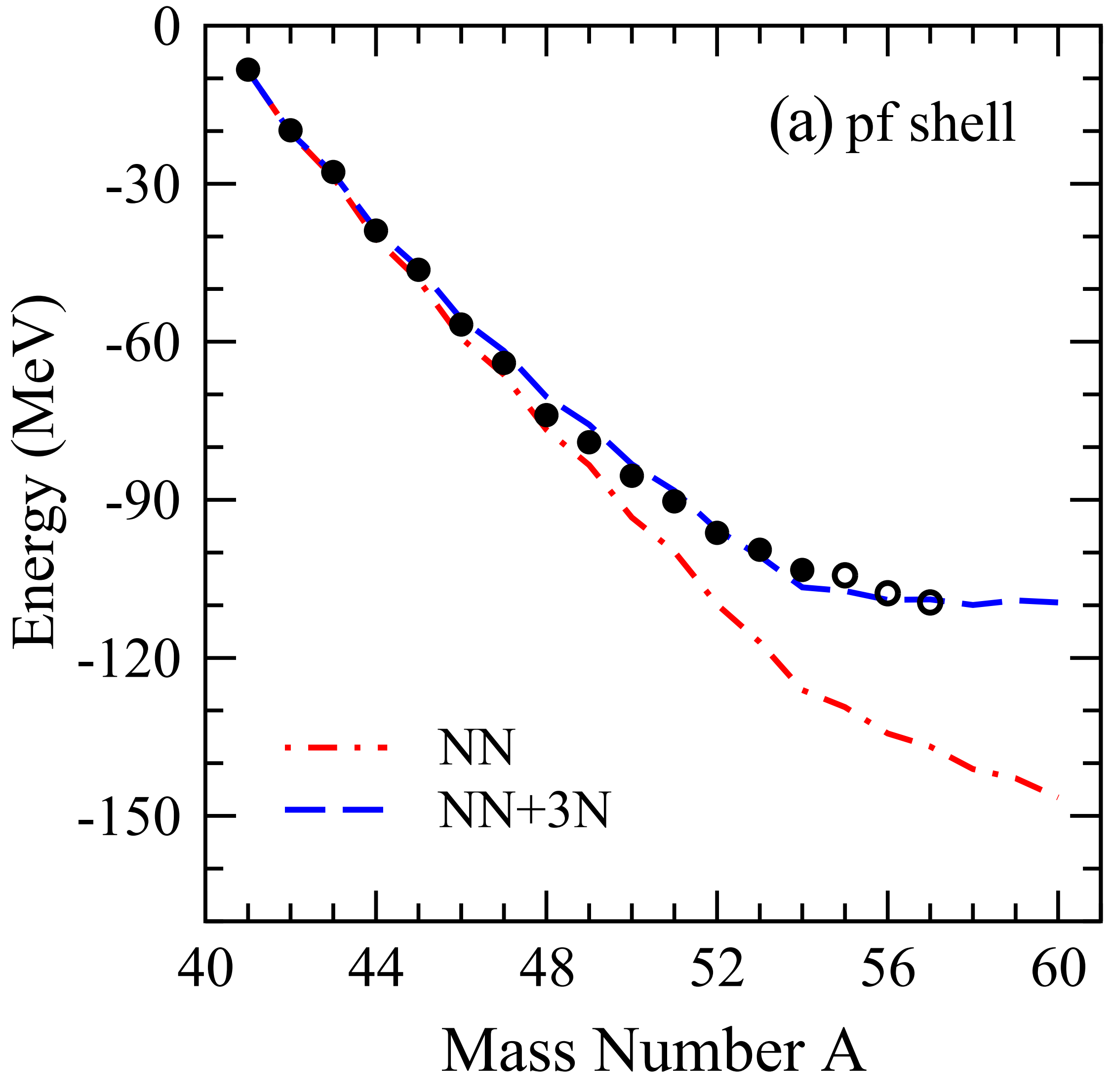
Gamow shell model

Ma +, PLB **802**, 135257 (2020)

≲ 10 MeV repulsion

3NF contributions to ground-state energies

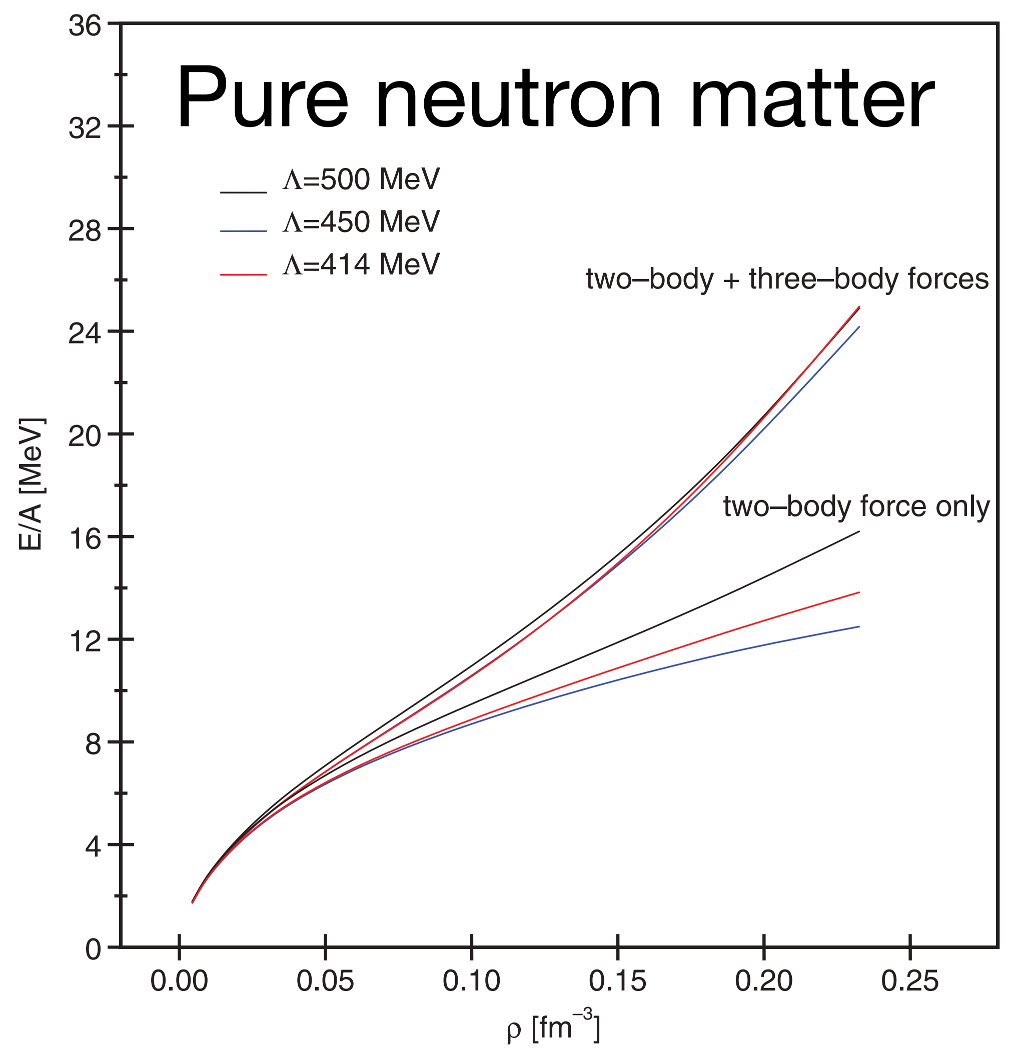
pf-shell nuclei

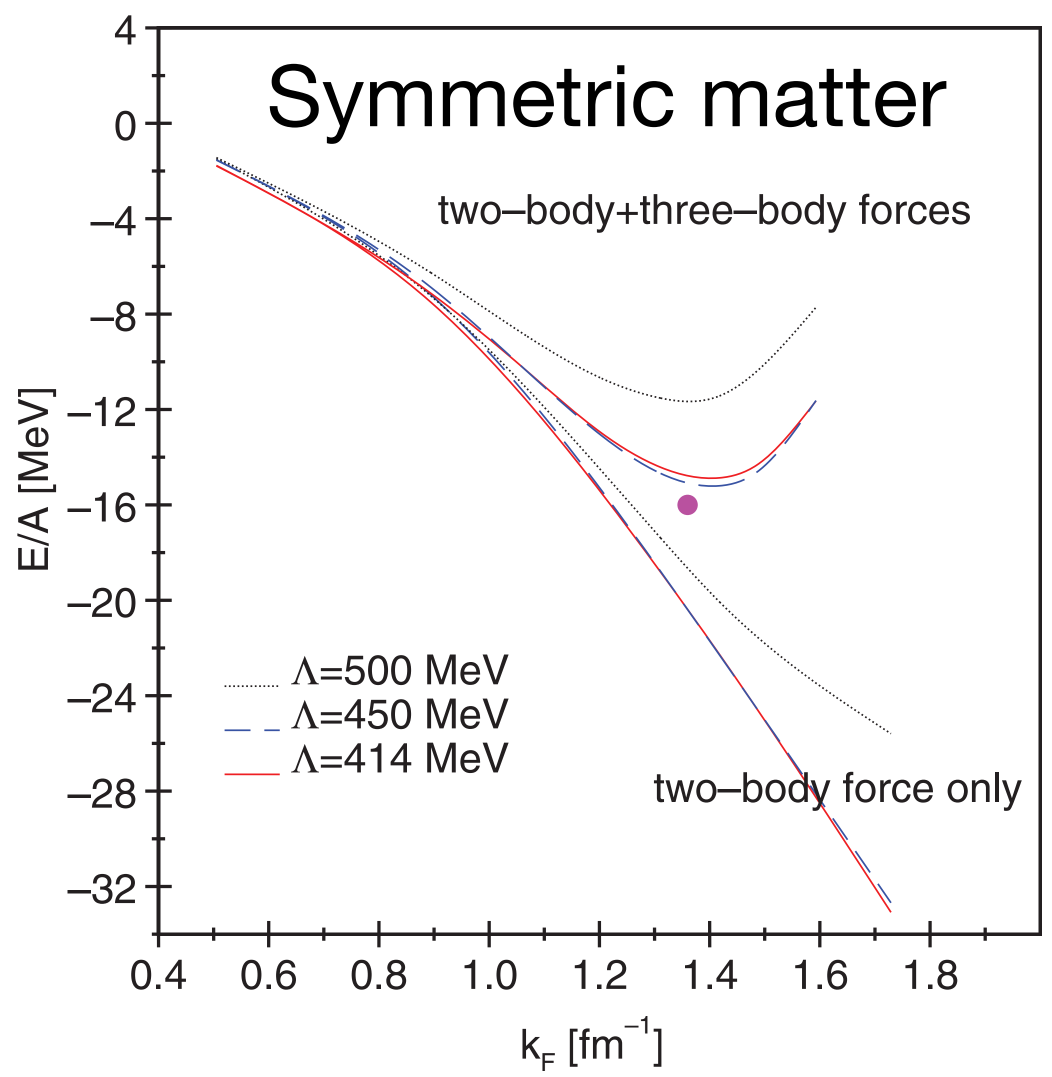


RSM Holt +, PRC **90**, 024312 (2014)

$\lesssim$ 10–20 MeV/A repulsion

Nuclear matter





Perturbative calc.

Coraggio +, PRC **87**, 014322 (2013)
Coraggio +, PRC **89**, 044321 (2014)

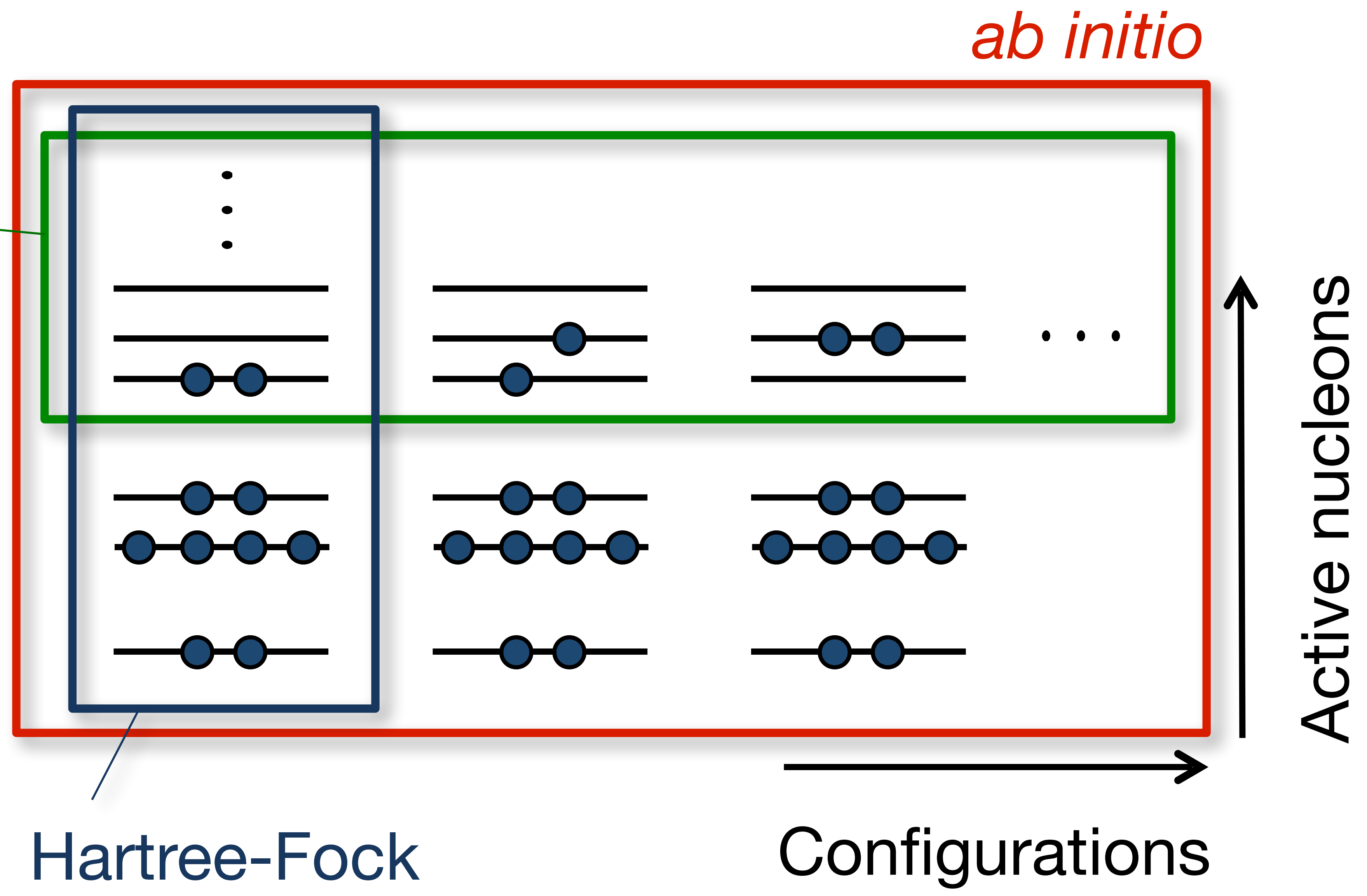
$\sim$ 10—20 MeV/A repulsion

Implementation of chiral forces to shell model

Valence-space diagonalization

Shell model

Diagonalization of Hamiltonian
within valence model space
using harmonic-oscillator bases



Realistic shell model (RSM)

= Shell model (valence space) with a realistic force

Applicable to heavier systems

Realistic Hamiltonian

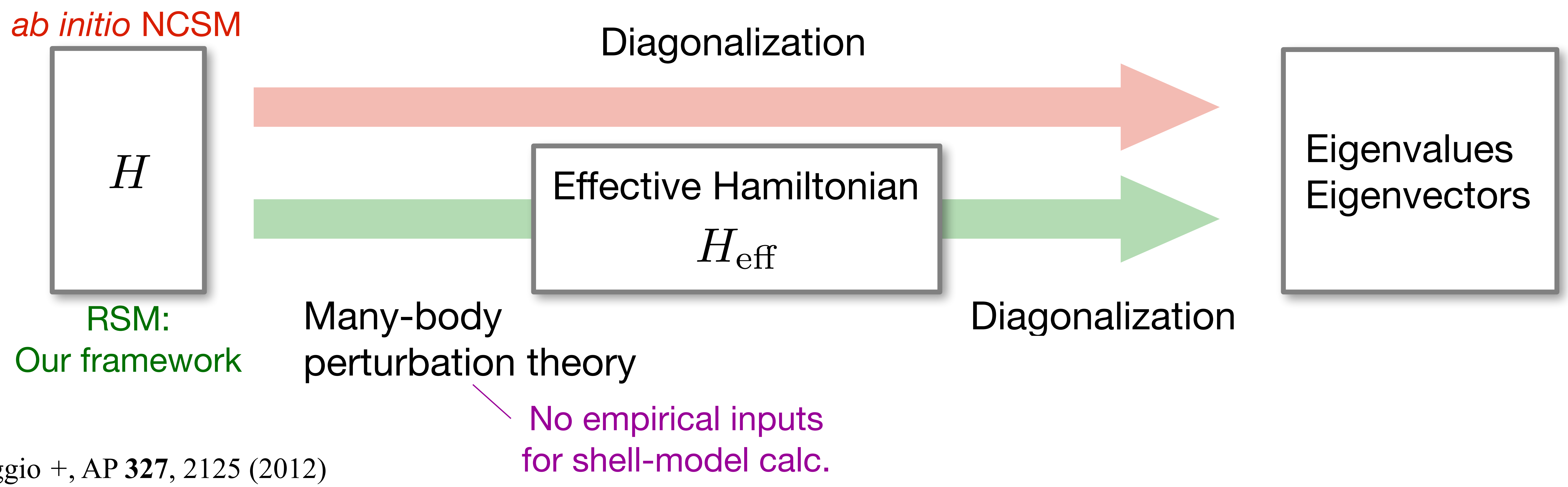
$$H = H_{1\mathrm{B}} + H_{2\mathrm{B}} + H_{3\mathrm{B}}$$

Single-particle energy

Chiral 2NF at N^{3}LO + Coulomb

Chiral 3NF at N^{2}LO

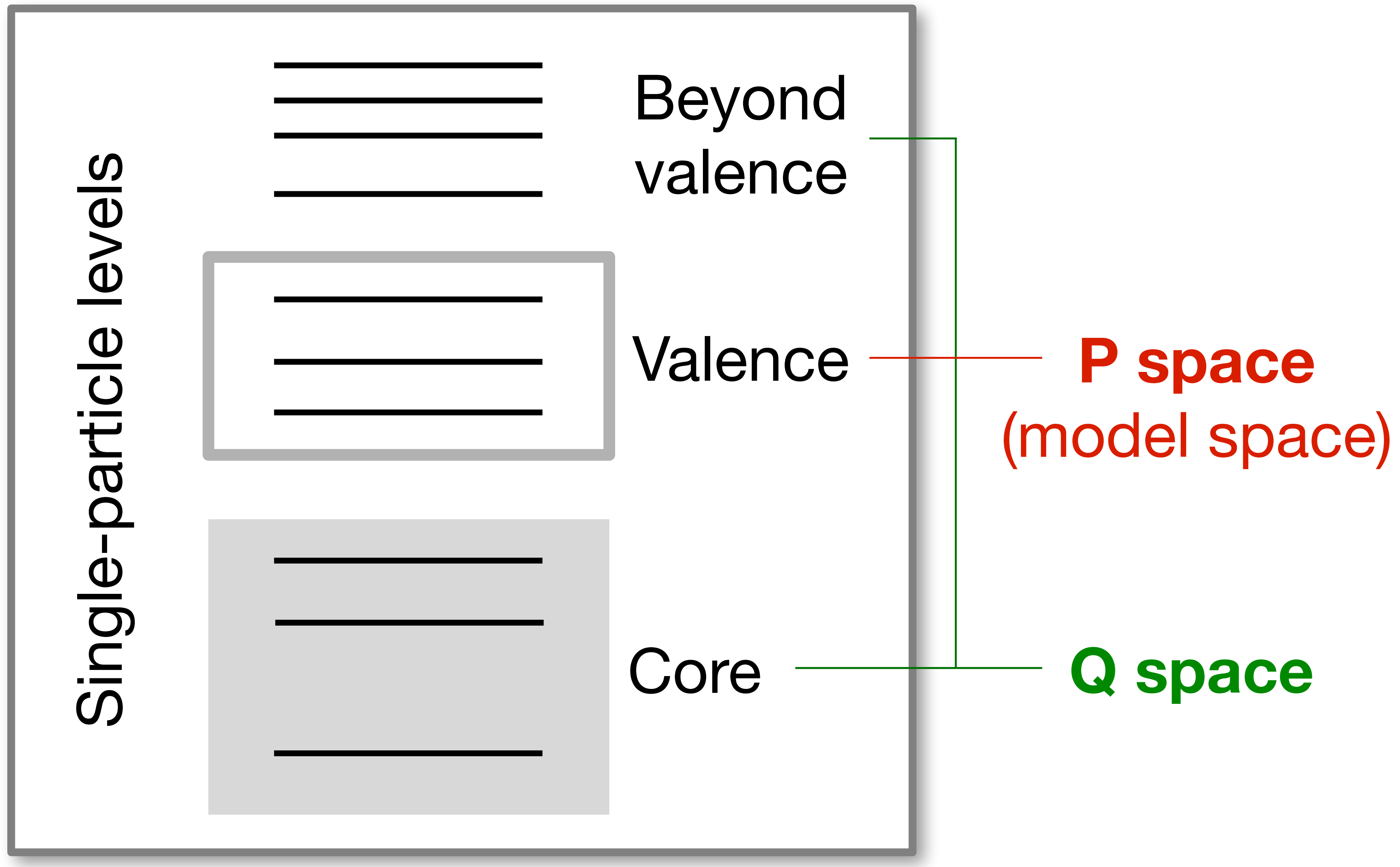
Shell-model framework



Coraggio +, AP **327**, 2125 (2012)

Many-body perturbation theory (1)

Coraggio +, AP **327**, 2125 (2012)



$$H = H_0 + H_1$$

$$H_{\text{eff}} = P\mathcal{H}P = PH_0P + H_1^{\text{eff}}$$

$$\mathcal{H} = X^{-1}HX$$

$$X = e^{\omega}$$

(Similarity transformation)

Feshbach projection operator

$$P = \sum_i |\Phi_i\rangle\langle\Phi_i|\,, \quad Q = 1 - P$$

$$P^2 = P, \quad Q^2 = Q, \quad PQ = QP = 0$$

$$\begin{array}{c|cc} & P & Q \\ \hline P & H_{\text{eff}} & 0 \\ Q & 0 & Q\mathcal{H}Q \end{array}$$

Many-body perturbation theory (2)

Coraggio +, AP **327**, 2125 (2012)

Suzuki & Lee, PTP **64**, 2091 (1980)

$$\omega = Q\omega P$$
$$P\omega P = Q\omega Q = P\omega Q = 0$$

$$\omega = \begin{pmatrix} P\omega P & P\omega Q \\ Q\omega P & Q\omega Q \end{pmatrix} \to \begin{pmatrix} 0 & 0 \\ Q\omega P & 0 \end{pmatrix}$$

$$P\mathcal{H}P = PHP + PHQ\omega$$
$$P\mathcal{H}Q = PHQ$$
$$Q\mathcal{H}P = QHP + QHQ\omega - \omega PHP - \omega PHQ\omega = 0$$
$$Q\mathcal{H}Q = QHQ - \omega PHQ$$

Decoupling equation

Many-body perturbation theory (3)

Coraggio +, AP **327**, 2125 (2012)

$$\omega = \frac{1}{\epsilon_0 - QHQ} QH_1P - \frac{1}{\epsilon_0 - QHQ} \omega H_1^{\text{eff}}$$

$\hat{Q}$ box

$$\hat{Q}(\epsilon_0) = PH_1P + PH_1Q \frac{1}{\epsilon_0 - QHQ} QH_1P$$

$$H_1^{\text{eff}}(\omega) = \hat{Q}(\epsilon_0) - PH_1Q \frac{1}{\epsilon_0 - QHQ} \omega H_1^{\text{eff}}(\omega)$$

$$= \left[1 + PH_1Q \frac{1}{\epsilon_0 - QHQ} \omega\right]^{-1} \hat{Q}(\epsilon_0)$$

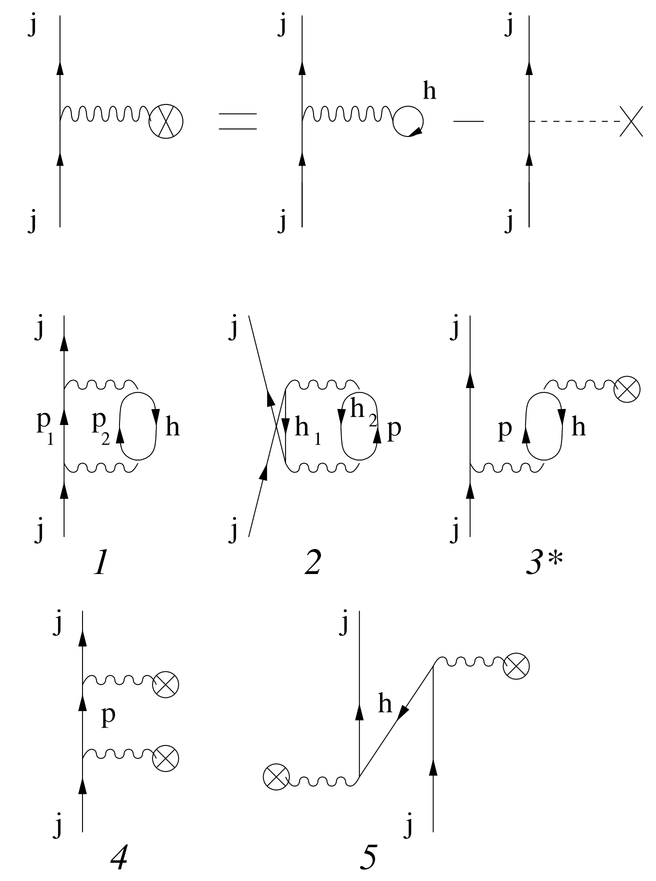
Many-body perturbation theory (4)

Coraggio +, AP **327**, 2125 (2012)

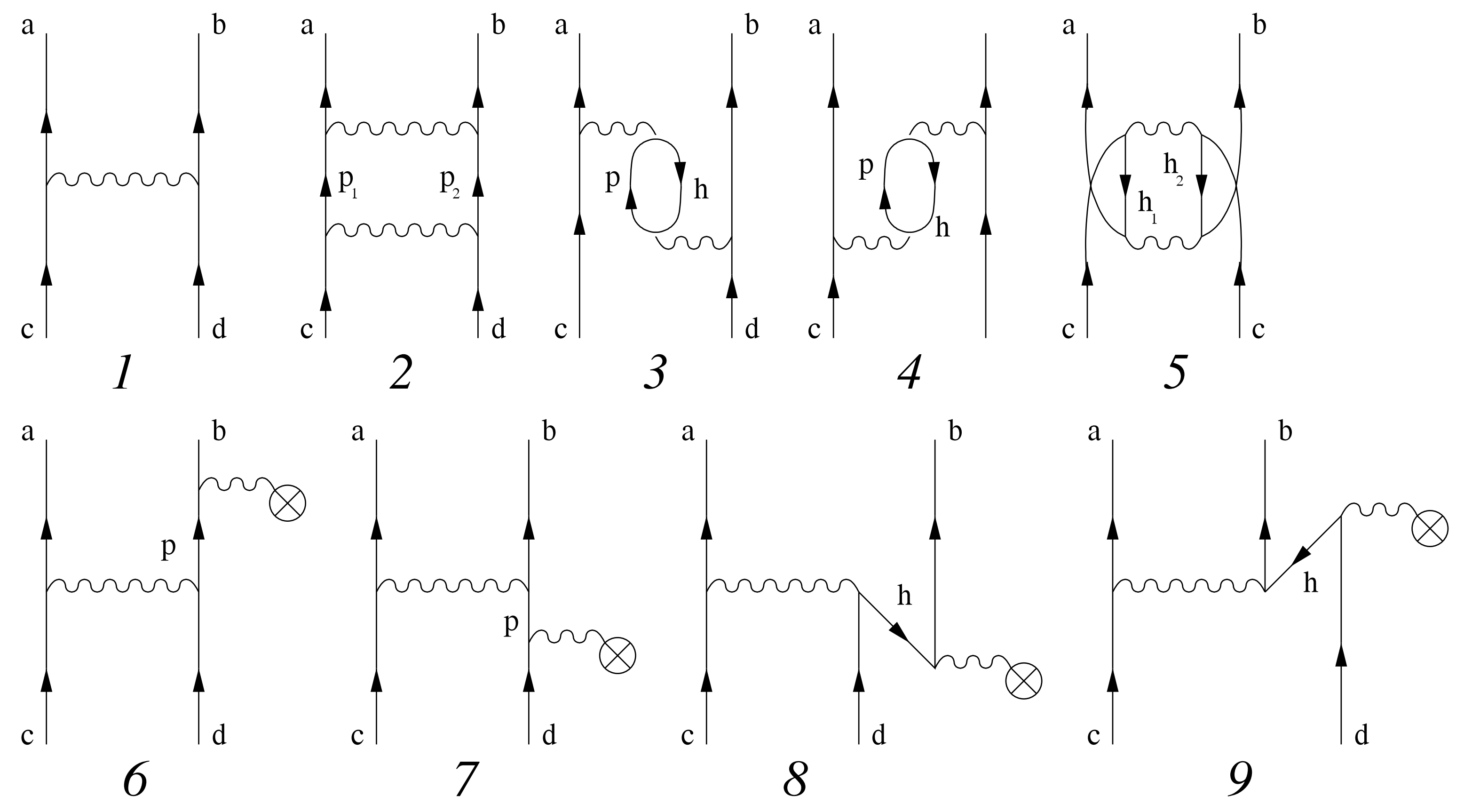
Q box $$\hat{Q}(\epsilon_0) = PH_1P + PH_1Q\frac{1}{\epsilon_0 - QHQ}QH_1P$$

→ Perturbative expansion $$\frac{1}{\epsilon_0 - QHQ} = \sum_{n=0}^{\infty} \frac{(QH_1Q)^n}{(\epsilon_0 - QH_0Q)^{n+1}}$$

Diagrammatic expansion (examples)



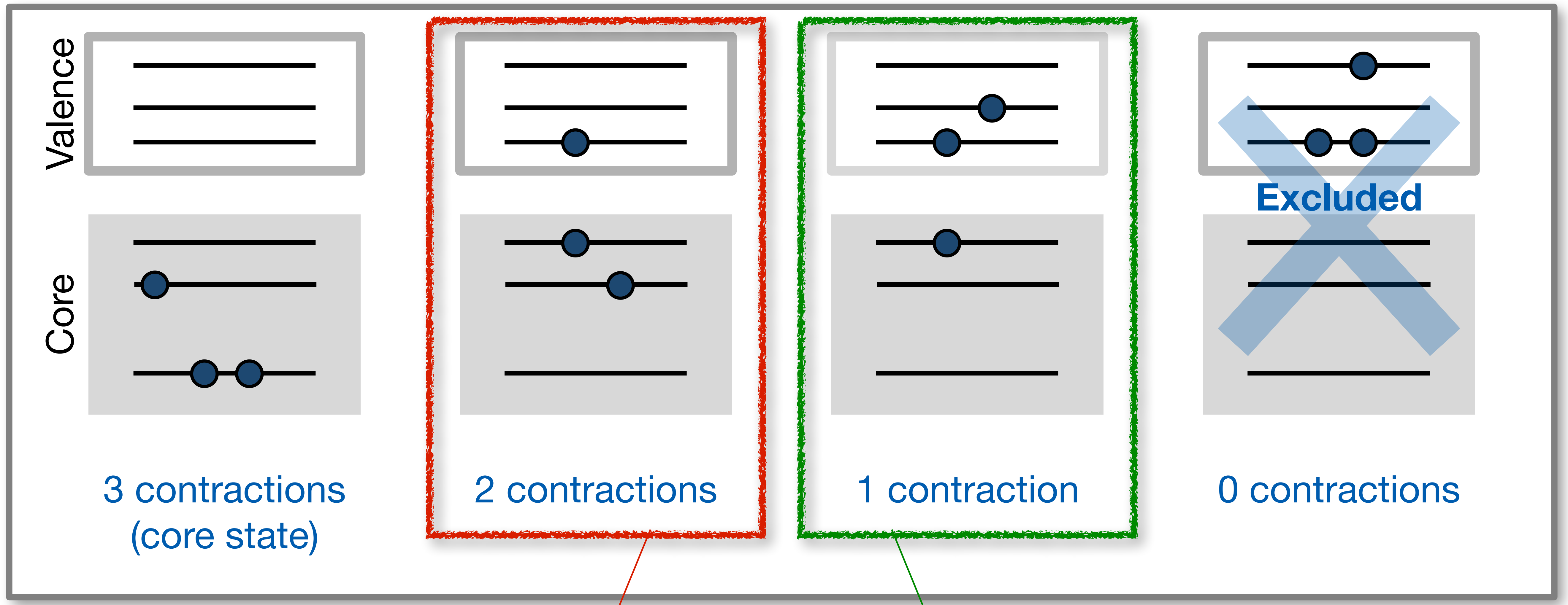
1-body diagrams



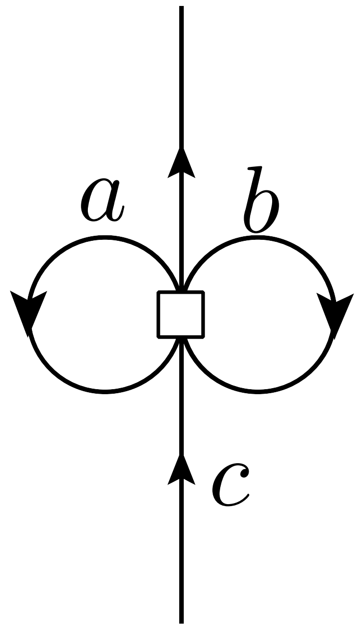
2-body diagrams

Normal-ordering approx. of 3NF

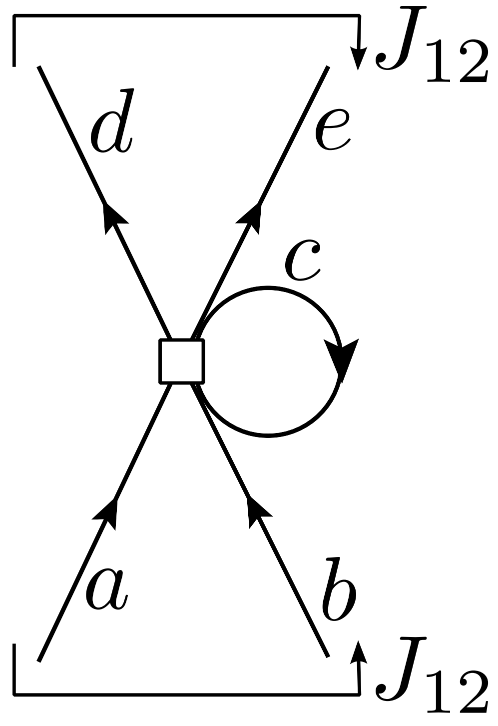
Normal-ordering approximation
→ 3NF as a modification of SPE and 2BMEs



Normal-ordered 1-body term



Normal-ordered 2-body term



Normal-ordering approx. of 3NF

Normal-ordering approximation

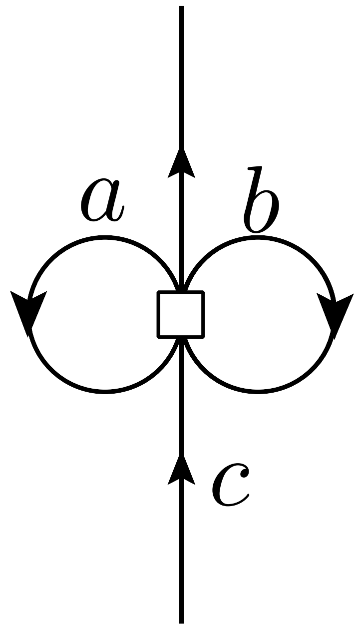
→ **3NF as a modification of SPE and 2BMEs**

$$abcfed \equiv a_a^\dagger a_b^\dagger a_c^\dagger a_f a_e a_d$$

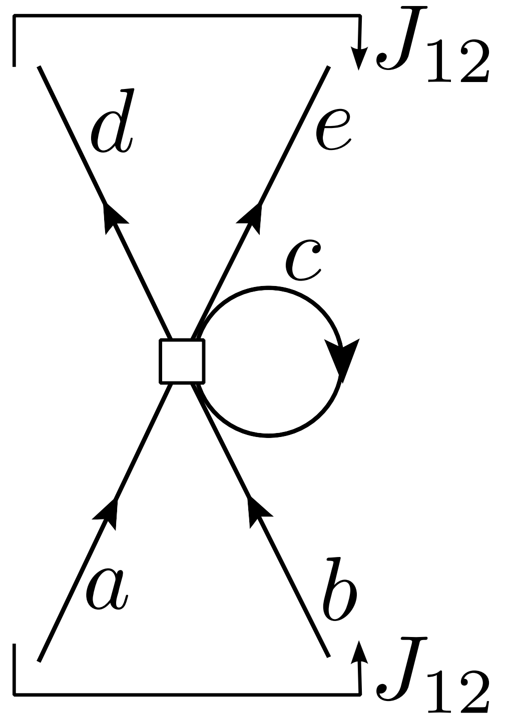
$$\begin{aligned} A^{(1\mathrm{B})}_{abc,def} = {} & \overbracket{af}\,\overbracket{bd} : ce : - \overbracket{af}\,\overbracket{be} : cd : + \overbracket{af}\,\overbracket{ce} : bd : - \overbracket{af}\,\overbracket{cd} : be : + \overbracket{ae}\,\overbracket{bf} : cd : - \overbracket{ae}\,\overbracket{bd} : cf : \\ & - \overbracket{ae}\,\overbracket{cf} : bd : - \overbracket{ae}\,\overbracket{cd} : bf : - \overbracket{ad}\,\overbracket{bf} : ce : - \overbracket{ad}\,\overbracket{be} : cf : + \overbracket{ad}\,\overbracket{cf} : be : - \overbracket{ad}\,\overbracket{ce} : bf : \\ & - \overbracket{bf}\,\overbracket{ce} : ad : + \overbracket{bf}\,\overbracket{cd} : ae : + \overbracket{be}\,\overbracket{cf} : ad : + \overbracket{be}\,\overbracket{cd} : af : - \overbracket{bd}\,\overbracket{cf} : ae : + \overbracket{bd}\,\overbracket{ce} : af :, \end{aligned}$$

$$\begin{aligned} A^{(2\mathrm{B})}_{abc,def} = {} & \overbracket{af} : bced : - \overbracket{ae} : bcfd : + \overbracket{ad} : bcfe : - \overbracket{bf} : aced : + \overbracket{be} : acfd : - \overbracket{bd} : acfe : \\ & + \overbracket{cf} : abed : - \overbracket{ce} : abfd : + \overbracket{cd} : abfe :, \end{aligned}$$

Normal-ordered 1-body term



Normal-ordered 2-body term



Results

Results

will be given in Seminar tomorrow